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Abstract

The technique of ANOVA has been widely used in Economics and Finance where the observations are usually time-dependent but the model itself is treated as independent in time. In this paper, we extend an ANOVA model by relaxing the assumption of independence in time. We further relax the assumption of homoskedasticity in the traditional profile analysis by introducing GARCH innovations in our proposed profile analysis that allows for both autoregressive and moving average components in the heteroskedastic variance to display a high degree of persistence. We reprise the model with regards to the issue of American Depository Receipts by relaxing the time dependence assumption that has been ignored in the literature. Applying our model, we find that the returns from the stocks and the American Depository Receipts are time dependent and hence the traditional ANOVA cannot fully explore the time effect from the data.

1. Introduction

The technique of ANOVA has been widely applied in Economics [17, 18] and Finance [26, 15]. However, in most studies, the observations measured in time are usually treated as independent in time though it is well-known that in many cases they are time-dependent [10, 13, 4]. In this paper, we extend an ANOVA model by relaxing the assumption of independence in time. We further relax the assumption of homoskedasticity in the traditional profile analysis by introducing GARCH innovations [5, 6, 3, 7] in our proposed profile analysis that allows for both autoregressive and moving average components in the heteroskedastic variance to display a high degree of persistence.

Assume that X_{ijk} is a T-dimensional multivariate-normal-distributed random vector in which its entry at time t follows:

$$X_{ijkt} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijkt} \tag{1}$$

for $i = 1, \ldots, a$; $j = 1, \ldots, b$; $k = 1, \ldots, c$; $t = 1, \ldots, T$; where μ is the T-dimensional vector of common effect; α_i, β_j and γ_k are the corresponding T-dimensional vectors of the treatment effects satisfying $\sum_{i=1}^{a} \alpha_i = \sum_{j=1}^{b} \beta_j = \sum_{k=1}^{c} \gamma_k = 0$; and ε_{ijk} follows a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (p, q) model such that

$$
\begin{aligned}
\varepsilon_{ijkt} &= z_{ijkt}\sqrt{h_{ijkt}} \\
z_{ijkt} &\sim iid\ N(0, 1) \\
h_{ijkt} &= w_{ijk} + \sum_{u=1}^{p} a_{ijk,u}\varepsilon_{ijk,t-u}^2 + \sum_{v=1}^{q} b_{ijk,v}h_{ijk,t-v}.
\end{aligned}
$$

Assumption of white noise for innovations is sometimes very unrealistic as very frequently we find in data serial correlation of disturbances across periods and temporal dependence in second order moments of disturbances. Since the appearance of both autocorrelation and heteroscedasticity are commonly occurred in many time-series settings, especially in financial time series, many studies suggest to use nonlinear

time series structures to model these features in the data. Among them, the Autoregressive Conditional Heteroskedasticiy (ARCH) of [12] and its generalized version GARCH of [5] have offered a powerful tool in investigating the conditional volatility or second moment of financial series.

From theoretical and mathematical points of view, the rapidly spreading popularity of GARCH model is due to its technical beauty and tractability. For example, [3] remarked that a major contribution of the GARCH literature is the finding that apparent changes in the volatility of economic time series may be predictable and result from a specific type of nonlinear dependence rather than exogenous structural changes in variables. The $\text{GARCH}(p, q)$ innovations utilized in this model allows both autoregressive and moving average components to be included in the heteroskedastic variance to display a high degree of persistence. It also enables researchers make better forecast and take care of cluster errors and nonlinearities. Thus, the $\text{GARCH}(p, q)$ model has been widely accepted and used in modelling volatility in financial time series [3, 11, 20, 22]. Our focus is on X_{ijkt} with the model setting displayed in (1). This is a typical three-way Multivariate analysis of variance (MANOVA) of one observation per cell with GARCH innovations. We assume that there is no interaction among α, β and γ effects since there is only one observation per cell. Nonetheless, we note that if $X_{ijk} = (X_{ijk1}, \ldots, X_{ijkT})'$ are the repeated measures of a subject from time 1 to time T, then $\mu_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k$ is the profile of the subject for the (i, j, k) treatment combination and thus interaction among α, β and γ effects could be included in the model.

If we have only one factor (say, α_i) with two groups ($a = 2$), then this is a typical profile analysis. The typical profile analysis has been considered and discussed in many textbooks, see for example, [23, 16]. In [27] the author extended profile analysis to several groups and in [21] the authors gave a review on this subject. However, the models being studied in the literature, so far, consist of only one factor which is further assumed to be independent of time. In this paper we extend the model by modifying it into a profile analysis model with three factors and further relax the assumption of time-independence.

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In the profile analysis, there are three hypotheses of interest. First, researchers are interested in testing the 'parallelism' hypothesis that there is no interaction between the response and any of the treatments; that is, the profiles are parallel. When the 'parallelism' hypothesis is accepted, our model enables academic and practitioners to further proceed to test the 'no treatment effect' hypothesis and/or the 'no trial effect' hypothesis. The former is to test whether there is no treatment effects in ANOVA (that is, to test whether these parallel profiles are, in fact, one profile) whereas the latter is to test whether these parallel profiles are, in fact, horizontal lines.

To demonstrate the superiority of our approach over the traditional ANOVA, in this paper we further compare the performances of our model and the traditional ANOVA model by analyzing the returns for the stocks and the American Depository Receipts. Readers may refer to [24, 28, 8, 9, 2] for the study of the stocks and American Depository Receipts. We find that our model successfully identifies all significant factors whereas, without consideration of the time dependent factor, the traditional ANOVA model fails to do likewise. This shows the traditional ANOVA cannot fully explore the time effect from the data whereas our model enables researchers to do so.

In Section 2, we will present the testing procedures for the above-mentioned three types of hypotheses. In Section 3, a real example to analyze the returns of the stocks and American Depository Receipts is illustrated to demonstrate the superiority of the testing procedures developed in Section 2. Section 4 will round up this paper with the indispensable conclusion.

2. Testing the three types of hypotheses

In this section, we first state three types of hypotheses to be tested by utilizing our model, namely the 'parallelism' hypothesis, the 'no treatment effect' hypothesis and the 'no trial effect' hypothesis. It is important to note that in conducting the analysis, the 'parallelism' hypothesis has to be tested first. If this hypothesis is not rejected, we then proceed to test the 'no treatment effect' hypothesis or the 'no trial effect' hypothesis. If the 'parallelism' hypothesis is rejected, it is meaningless to conduct tests for the 'no treatment effect' or the 'no trial effect' hypotheses.

Recall that the T-dimensional vector x_{ijk} with each of its entries defined in (1) follows a multivariate normal distribution with mean $\mu_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k$ and covariance matrix Σ such that

$$x_{ijk} = \begin{pmatrix} x_{ijk1} \\ \vdots \\ x_{ijkT} \end{pmatrix} \sim N_T(\mu_{ijk}, \Sigma), \tag{2}$$

where $\mu_{ijk} = \begin{pmatrix} \mu_{ijk1} \\ \vdots \\ \mu_{ijkT} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_T \end{pmatrix} + \begin{pmatrix} \alpha_{i1} \\ \vdots \\ \alpha_{iT} \end{pmatrix} + \begin{pmatrix} \beta_{j1} \\ \vdots \\ \beta_{jT} \end{pmatrix} + \begin{pmatrix} \gamma_{k1} \\ \vdots \\ \gamma_{kT} \end{pmatrix}.$

Now, we are ready to describe the three hypotheses as in the following subsections.

2.1. The parallelism hypothesis

We first study the parallelism hypothesis. To do that, we define a $(T-1) \times T$ constant matrix D and a $T-1$ vector Y_{ijk} such that:

$$D = \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 1 & -1 \end{pmatrix}$$

and

$$Y_{ijk} = \begin{pmatrix} y_{ijk,1} \\ \vdots \\ y_{ijk,T-1} \end{pmatrix} = \begin{pmatrix} x_{ijk,1} - x_{ijk,2} \\ \vdots \\ x_{ijk,T-1} - x_{ijk,T} \end{pmatrix} = DX_{ijk}.$$

Then, for any fixed indices j and k, the 'parallelism' hypothesis of α-profile is $H_1^\alpha : D\mu_{1jk} = \dots = D\mu_{ajk}$. Similarly, for any fixed indices i and k, the 'parallelism' hypothesis of β-profile is $H_1^\beta : D\mu_{i1k} = \dots = D\mu_{ibk}$ whereas for any fixed indices i and j, the 'parallelism' hypothesis of γ-profile is $H_1^\gamma : D\mu_{ij1} = \dots = D\mu_{ijc}$.

Before stating the test statistics for testing these hypotheses, herewith we first

define the $(T-1) \times (T-1)$ sum-of-squares-and-cross-product (SSCP) matrices as follows:

$$
\begin{aligned}
V &= \sum_i^a \sum_j^b \sum_k^c (Y_{ijk} - \bar{Y}_{...})(Y_{ijk} - \bar{Y}_{...})' , \\
A &= \sum_i^a \sum_j^b \sum_k^c (\bar{Y}_{i..} - \bar{Y}_{...})(\bar{Y}_{i..} - \bar{Y}_{...})' = bc \sum_i^a (\bar{Y}_{i..} - \bar{Y}_{...})(\bar{Y}_{i..} - \bar{Y}_{...})' , \\
B &= \sum_i^a \sum_j^b \sum_k^c (\bar{Y}_{.j.} - \bar{Y}_{...})(\bar{Y}_{.j.} - \bar{Y}_{...})' = ac \sum_j^b (\bar{Y}_{.j.} - \bar{Y}_{...})(\bar{Y}_{.j.} - \bar{Y}_{...})' , \\
C &= \sum_i^a \sum_j^b \sum_k^c (\bar{Y}_{..k} - \bar{Y}_{...})(\bar{Y}_{..k} - \bar{Y}_{...})' = ab \sum_k^c (\bar{Y}_{..k} - \bar{Y}_{...})(\bar{Y}_{..k} - \bar{Y}_{...})' , \\
E &= V - A - B - C ,
\end{aligned}
$$

where

$$
\bar{Y}_{...} = \frac{1}{abc} \sum_i^a \sum_j^b \sum_k^c Y_{ijk} , \qquad \bar{Y}_{i..} = \frac{1}{bc} \sum_j^b \sum_k^c Y_{ijk} ,
$$

$$
\bar{Y}_{.j.} = \frac{1}{ac} \sum_i^a \sum_k^c Y_{ijk} , \qquad \bar{Y}_{..k} = \frac{1}{ab} \sum_i^a \sum_j^b Y_{ijk} .
$$

From the theory of multivariate distribution analysis, we have

$$
Y_{ijk} \sim N_{T-1}(D\mu_{ijk}, D\Sigma D') .
$$

This is precisely a Multivariate Analysis of Variance (MANOVA) setting on Y_{ijk} with one observation per cell. The matrices V and E are the total SSCP matrix and the error SSCP matrix respectively; A, B and C are the hypothesized SSCP matrices for testing the corresponding α_i, β_j and γ_k effects. These SSCP matrices follow a Wishart distribution with scale matrix Σ and various degrees of freedom. The ANOVA in table form can be expressed as follows:

SSCP matrix	Degrees of freedom	Wishart distribution
A	a-1	$W_{T-1}(a-1,\Sigma)$
B	b-1	$W_{T-1}(b-1,\Sigma)$
C	c-1	$W_{T-1}(c-1,\Sigma)$
E	e	$W_{T-1}(e,\Sigma)$
V	abc-1	$W_{T-1}(abc-1,\Sigma)$

where $e = abc - a - b - c + 2$. There are many test statistics could be used to analyze the effects in MANOVA. Among them, we apply Wilk's Lambda test statistic and utilize Barlett's large sample approximation to obtain its distribution as shown in the following.[1]

1. A test of $H_1^\alpha : D\mu_{1jk} = \cdots = D\mu_{ajk}$ versus its alternative $K_1^\alpha : D\mu_{ijk} \neq D\mu_{i'jk}$ for some i and i' could be conducted by employing

$$\Lambda_\alpha = \frac{|E|}{|E+A|} \sim \Lambda_{T-1}(e, a-1) ,\tag{3}$$

in which its test statistic

$$-[e - \frac{1}{2}(T - a + 1)]\log \Lambda_\alpha \tag{4}$$

is approximately distributed as Chi-Square with $(T-1)(a-1)$ degree of freedom. If $a = 2$, one could further derive an exact F test statistic to test H_1^α by

$$F = \frac{e - T + 2}{|a - T| + 1} \times \frac{1 - \Lambda_\alpha}{\Lambda_\alpha} . \tag{5}$$

This test statistic has a F distribution with $|a - T + 1| + 1$ and $e - T + 2$ degrees of freedom under H_1^α.

---

[1]More specifically, we let W_1 and W_2 be two $p \times p$ independent random matrices having Wishart distribution with scale matrix Σ and degrees of freedom ν_1 and ν_2 respectively, denoted by $W_1 \sim W_p(\nu_1, \Sigma)$ and $W_2 \sim W_p(\nu_2, \Sigma)$. Then, $\Lambda = \frac{|W_1|}{|W_1+W_2|}$ has a Wilk's Lambda distribution, denoted by $\Lambda \sim \Lambda_p(\nu_1, \nu_2)$. Applying the Barlett's large sample approximation to Wilk's Lambda test statistic Λ which is $-[\nu_1 - \frac{1}{2}(p - \nu_2 + 1)]\log \Lambda$, one can easily show that the distribution of the test statistic is approximately distributed as a Chi-Square distribution with $p\nu_2$ degree of freedom (See Section 6.4 of [16] for details).

2. Similarly, a test of $H_1^\beta : D\mu_{i1k} = \cdots = D\mu_{ibk}$ versus its alternative $K_1^\beta :$ $D\mu_{ijk} \neq D\mu_{ij'k}$ for some j and j' could be conducted by using

$$\Lambda_\beta = \frac{|E|}{|E + B|} \sim \Lambda_{T-1}(e, b - 1) , \tag{6}$$

in which its test statistic

$$-[e - \frac{1}{2}(T - b + 1)] \log \Lambda_\beta \tag{7}$$

is approximately distributed as Chi-Square with $(T-1)(b-1)$ degree of freedom. If $b = 2$, we will then obtain an exact F test to test H_1^β by utlizing

$$F = \frac{e - T + 2}{|b - T| + 1} \times \frac{1 - \Lambda_\beta}{\Lambda_\beta} . \tag{8}$$

This test statistic has a F distribution with $|b - T + 1| + 1$ and $e - T + 2$ degrees of freedom under H_1^β. In addition,

3. a test of $H_1^\gamma : D\mu_{ij1} = \cdots = D\mu_{ijc}$ versus its alternative $K_1^\gamma : D\mu_{ijk} \neq D\mu_{ijk'}$ for some k and k' could be conducted by adopting

$$\Lambda_\gamma = \frac{|E|}{|E + C|} \sim \Lambda_{T-1}(e, c - 1) , \tag{9}$$

in which its test statistic

$$-[e - \frac{1}{2}(T - c + 1)] \log \Lambda_\gamma \tag{10}$$

is approximately distributed as Chi-Square with $(T-1)(c-1)$ degree of freedom. If $c = 2$, an exact F test such that

$$F = \frac{e - T + 2}{|c - T| + 1} \times \frac{1 - \Lambda_\gamma}{\Lambda_\gamma} . \tag{11}$$

can be easily derived to test the null hypothesis H_1^γ. This test statistic has a F distribution with $|c - T + 1| + 1$ and $e - T + 2$ degrees of freedom under H_1^γ.

2.2. The no treatment effect hypothesis

If the 'parallelism' hypothesis is not rejected, one could proceed to test whether there exists any α_i, β_j or γ_k treatment effect. In other words, we are testing if the parallel profiles are, in fact, one profile. For example, when H_1^α is not rejected, we can then test $H_2^\alpha : 1_T'\mu_{1jk} = \cdots = 1_T'\mu_{ajk}$ versus $K_2^\alpha : 1_T'\mu_{ijk} \neq 1_T'\mu_{i'jk}$ for some i and i' where 1_T is a $T \times 1$ vector of ones. The reason is that if μ_{ijk} and $\mu_{i'jk}$ are parallel, then they differ by an unknown constant $\delta_{i,i'}$ due to the different treatment groups i and i'. Under H_2^α, the sums of the components from μ_{ijk} and $\mu_{i'jk}$ are equal, implying that $\delta_{i,i'}$ is equal to zero or these parallel profiles are coincided. Applying similar reasoning, one will draw similar inferences to testing β-profiles and γ-profiles. Thus, the 'no treatment effect' hypotheses could be formulated in the following:

1. $H_2^\alpha : 1_T'\mu_{1jk} = \cdots = 1_T'\mu_{ajk}$; (that is, no α effects),

2. $H_2^\beta : 1_T'\mu_{i1k} = \cdots = 1_T'\mu_{ibk}$; (that is, no β effects), and

3. $H_2^\gamma : 1_T'\mu_{ij1} = \cdots = 1_T'\mu_{ijc}$; (that is, no γ effects).

Defining $z_{ijk} = \sum_{t=1}^T X_{ijkt} = 1_T'X_{ijk}$, we have $z_{ijk} \sim N(1_T'\mu_{ijk}, 1_T'\Sigma 1_T)$. Note that testing H_2^α, H_2^β or H_2^γ is equivalent to testing the treatment effects of a three-way ANOVA model on z_{ijk}. Thus, the F-tests of ordinary three-way ANOVA on z_{ijk} could then be employed to test these hypotheses.

2.3. The no trial effect hypothesis

On the condition that the 'parallelism' hypothesis is not rejected, one could further proceed to test if these parallel profiles are, in fact, horizontal lines. For example, given the hypothesis that $H_1^\alpha : D\mu_{1jk} = \cdots = D\mu_{ajk}$ is not rejected for any fixed indices j and k, the no trial effect hypothesis can be formulated as $H_3^\alpha : D\theta_{jk} = 0$ versus $K_3^\alpha : D\theta_{jk} \neq 0$, where θ_{jk} is denoted to be the common $\mu_{1jk} = \cdots = \mu_{ajk}$ in H_1^α. The reason is that if $H_1^\alpha : D\mu_{1jk} = \cdots = D\mu_{ajk} = D\theta_{jk}$ is not rejected, $H_3^\alpha : D\theta_{jk} = 0$ implies $D\mu = 0$. That is, these α-profiles are, in fact, horizontal lines. In order to test

H_3^α, we define $\bar{X}_{.jk} = (1/a) \sum_{i=1}^{a} X_{ijk}$ (note that $D\bar{X}_{.jk} \sim N_{T-1}(D\theta_{jk}, (1/a)D\Sigma D')$). We can then apply Hotelling's T^2 test based on $\bar{X}_{.jk}$ to test $H_3^\alpha : \theta_{jk} = 0$. Thereafter, the no trial effect hypotheses can then be formulated as follows:

1. Given that $H_1^\alpha : D\mu_{1jk} = \cdots = D\mu_{ajk} = D\theta_{jk}$, a test of $H_3^\alpha : D\theta_{jk} = 0$ versus $K_3^\alpha : D\theta_{jk} \neq 0$ could be conducted by using

$$T_\alpha^2 = a(D\bar{X}_{.jk})'(DED')^{-1}(D\bar{X}_{.jk}) \tag{12}$$

where $(a - T + 1)T_\alpha^2/((T-1)(a-1))$ has a F-distribution with $T - 1$ and $e - T + 1$ degrees of freedom under H_3^α.

2. Given that $H_2^\beta : D\mu_{i1k} = \cdots = D\mu_{ibk} = D\theta_{ik}$, a test of $H_3^\beta : D\theta_{ik} = 0$ versus $K_3^\beta : D\theta_{ik} \neq 0$ could be conducted by utilizing

$$T_\beta^2 = b(D\bar{X}_{i.k})'(DED')^{-1}(D\bar{X}_{i.k}) . \tag{13}$$

One could easily show that the test statistic $(b - T + 1)T_\beta^2/((T-1)(b-1))$ follows a F-distribution with $T - 1$ and $e - T + 1$ degrees of freedom under H_3^β, where $\bar{X}_{i.k} = (1/b) \sum_{j=1}^{b} X_{ijk}$.

3. Given that $H_2^\gamma : D\mu_{ij1} = \cdots = D\mu_{ijc} = D\theta_{ij}$, a test of $H_3^\gamma : D\theta_{ij} = 0$ versus $K_3^\gamma : D\theta_{ij} \neq 0$ could then be conducted by employing

$$T_\gamma^2 = c(D\bar{X}_{ij.})'(DED')^{-1}(D\bar{X}_{ij.}) . \tag{14}$$

Similarly, one could easily verify that the test statistic $(c - T + 1)T_\gamma^2/((T-1)(c-1))$ follows a F-distribution with $T - 1$ and $e - T + 1$ degrees of freedom under H_3^γ, where $\bar{X}_{ij.} = (1/c) \sum_{k=1}^{c} X_{ij.}$.

3. Illustration

Numerous studies, for example see, [14, 19, 1], have demonstrated that the risk of a portfolio that comprises purely of domestic shares can be significantly reduced by

adopting international diversification. Since the introduction of American Depository Receipt (ADR), a number of research studies, for example see [24, 28, 2], have further shown that international diversification can be efficiently achieved by investing in ADRs issued by foreign companies.

There are many studies on the underlying factors affecting the performance of ADRs. For example, [25] observed that Japanese ADRs have a higher mean return than underlying stocks, attributing to the appreciation of Japanese Yen's exchange rate against the US Dollar. In [9], the authors examined the determinants of ADR returns and their implications on international diversification and market segmentation by running cross-sectional regression analysis. They considered firm specific, industry and market factors as determinants of ADRs and underlying stock returns. In [8], the authors found significant differences in country and industry representations between the ADR and the corresponding world market portfolio. However, all of these studies do not take into consideration the time dependent factor in their model estimation.

To fill in the gap of the literature by relaxing the time independence assumption, we apply our model in (1) and the corresponding tests developed in Section 2 to study the issue of ADR. Yearly averages of daily returns (log-returns) of two types of securities from 1990 to 2000 are used in this study in which the two types of securities are ADR and the corresponding US\$ converted underlying stocks. For each type of security, we construct an array of portfolios of 9 different proportions of their portfolios in the domestic market (S&P500 index) and 8 different sizes of foreign securities.

Thereafter, we utilize our model in (1) with the corresponding three factors; namely, α=type of securities ($a = 2$), β=proportion of domestic stock ($b = 9$) and γ=size of foreign securities ($c = 8$) with $T = 11$ (11 yearly averages). We further conduct the test of the parallelism hypothesis as described in Section 2. The test statistics for H_1^α, H_1^β and H_1^γ are $\Lambda_\alpha = 0.28$, $\Lambda_\beta = 0.00005$ and $\Lambda_\gamma = 0.00094$ respectively. In testing H_1^α, we have an exact F test since $a = 2$. Employing (5), we obtain $F = 30.34$ which is F-distributed with 10 and 118 degrees of freedom under H_1^α. For testing H_1^β and H_1^γ, we use equations (6) and (9) with their approximate chi-square tests. As all

Table 1: Three-factor ANOVA table without time dependent factor

Source	DF	SS	MS	F	P
Type	1	0.000030	0.000030	0.01	0.932
Weight	8	0.271512	0.033939	8.40	0.000
Size	7	0.000605	0.000086	0.02	1.000
Type*Weight	8	0.000012	0.000001	0.00	1.000
Type*Size	7	0.000122	0.000017	0.00	1.000
Weight*Size	56	0.000234	0.000004	0.00	1.000
Type*Weight*Size	56	0.000047	0.000001	0.00	1.000
Error	1440	5.818007	0.004040	0.00	1.000
Total	1583	6.090569			

the p-values are 0, all these profiles are inferred not to be parallel and thus it is not necessary to proceed further to test the 'no treatment effect' hypothesis or the 'no trial effect' hypothesis.

Note that all the tests described in Section 2 can be easily performed by utilizing standard statistical packages such as SAS, SPSS and MINITAB with a suitable transformation on X_{ijk}. In testing the 'parallelism' hypothesis, we only need to conduct a MANOVA on $Y_{ijk} = DX_{ijk}$. For testing the 'no treatment effect' hypothesis, we only need to perform an ANOVA on $z_{ijk} = 1'_T X_{ijk}$. Similarly, as for the 'no trial effect' hypothesis, we only need to conduct a Hotelling T^2 test given in equations (12), (13) and (14).

In order to compare our model with an ordinary ANOVA model without the time dependent factor, we analyze the data using a balanced three-factor ANOVA. The three factors are the same as in our previous model but we have 11 replicates for each combination of each factor level. Then, there are a total of $2 \times 9 \times 8 \times 11 = 1584$ observations. Since there are replicates, we can fit an ANOVA model with all the interaction terms to be included. Table 1 gives the ANOVA table for this three-factor ANOVA without time dependent factor. All the effects are not significantly different from 0 except for the weight effect, inferring that, different from our findings by utilizing our model in (1), this traditional ANOVA model clearly fails to identify

all the significant factors and their interactions except weight alone. This shows that our model is superior.

4. Conclusions

In this paper, we develop the ANOVA model where the assumption of independence in time is relaxed. We also extend the traditional one-factor profile model to include three factors. Thereafter, we develop several test statistics in the proposed model. The first statistic is to test whether there is no interaction between the response and the treatments. When this hypothesis is not rejected, our other statistics could then be conducted to find out whether there is no treatment effects, and whether these parallel profiles are in fact horizontal lines.

The model that we have developed is illustrated by comparing the returns from the stocks and the American Depository Receipts. Utilizing our proposed model, we conclude that the returns of both stocks and American Depository Receipts are time-dependent. On the other hand, the traditional ANOVA cannot fully explore the time effect from the data. This shows the superiority and applicability of our proposed model over the traditional one.

Our model so far only extends the profile analysis to include three factors. Nonetheless, the model can easily be extended to include more factors and their interactions. Further research could also include to capture effects of these factors in the conditional variance equation of the GARCH model by the introduction of these factors and their interactions as exogenous variables in the conditional volatility.

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