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KANT ON EUCLID: GEOMETRY IN PERSPECTIVE

I. The Perspectival Aim of the first Critique

There is a common assumption among philosophers, shared even by many Kant-scholars, that Kant had a naive faith in the absolute validity of Euclidean geometry, Aristotelian logic, and Newtonian physics, and that his primary goal in the Critique of Pure Reason was to provide a rational foundation upon which these classical scientific theories could be based. This, it might be thought, is the essence of his attempt to solve the problem which, as he says in a footnote to the second edition Preface, "still remains a scandal to philosophy and to human reason in general"--namely, "that the existence of things outside us...must be accepted merely on faith, and that if anyone thinks good to doubt their existence, we are unable to counter his doubts by any satisfactory proof" [K2:xxxix].¹ This assumption, in turn, is frequently used to deny the validity of some or all of Kant's philosophical project--or at least its relevance to modern philosophical understandings of scientific knowledge. Swinburne, for instance, asserts that an acceptance of the views expressed in Kant's first Critique "would rule out in advance most of the great achievements of science since his day."² Such assertions--and in particular the claim that Kant's argument for the transcendental ideality of space in the "Transcendental Aesthetic" of the first Critique can be ignored because it rests on an undefended acceptance of Euclidean geometry --are such a commonplace that there is no need to quote further examples.

But is the "satisfactory proof" Kant offers in the first Critique inextricably tied to the necessary validity of Euclidean geometry, Aristotelian logic, and Newtonian physics, as is so often assumed? A deeper understanding of both the structure and the purpose of Kant's argument, as well as a proper recognition of the context in which he employs such traditional theories, may reveal quite a different picture. In this article I will begin the task of setting right a travesty, on the basis of which Kant's first Critique has so often been rejected prematurely, by carefully examining the function of Kant's treatment of Euclidean geometry in his theoretical system. In short, I will argue that, far from assuming a kind of absolute validity for the

classical theories of Euclid, Aristotle, and Newton, Kant ties their views to a well-defined perspective in such a way that their validity is actually limited, and that in so doing he actually prepared the way (sometimes with surprising foresight) for modern developments in geometry, logic, and physics.

Throughout this article I will utilize a method of interpreting Kant's Critical System which I have developed in detail elsewhere [see note 3]. The key to this method is the recognition of the ubiquitous influence of the "principle of perspective" in Kant's thought, as it is expressed both in the overall structure of his System and in the myriad of distinctions within that System. This principle requires the philosopher to consider the general presuppositions implied by the context of any given question or problem, together with the rules that guide one to a solution, before attempting to deal with it. It implies that different, though equally valid, answers can be given to the same question if different "perspectives" are assumed. (Kant himself never explicitly states this as a principle, nor does he use the term "perspective" in any technical sense. However, he does make frequent use of a number of terms, such as "standpoint", "point of view", "way of thinking", etc., in precisely the way indicated by my use of the term "perspective".) Kant's three "general perspectives", or standpoints, are the theoretical, practical, and judicial, and each is developed, respectively, in one of his three Critiques. But the subject-matter elaborated from any one of these standpoints can itself be considered from several different angles, depending on the assumptions made at a given stage in its development. Accordingly, within each standpoint--and in this article I will be dealing only with the theoretical standpoint--Kant develops four subordinate perspectives: the transcendental, logical, empirical, and hypothetical. The "Transcendental Doctrine of Elements" in the first Critique has four main subdivisions, which correspond, respectively, to these four perspectives: the "Aesthetic", the "Analytic of Concepts", the "Analytic of Principles", and the "Dialectic".³ With this perspectival framework in mind, let us now turn to the question at hand and attempt to discern Kant's actual intentions vis-a-vis the above-mentioned classical doctrines, paying special attention to his view of Euclidean geometry.

II. The Transcendental Character of Geometry

In the second edition Preface to the first Critique, Kant says his "Copernican revolution" proceeds "in accordance with the example set by the geometers and physicists" [K2:xxii]. Their example (together with the logicians [viii-ix] is appealing because in Kant's opinion logic, mathematics and physics have all "entered upon the secure path of a science" [vii], precisely because they "have to determine their objects a priori" [x]--an insight which may seem strange to us, since "science" is now normally considered to be primarily empirical, but which is crucial in understanding Kant's intentions. Kant clearly regards the established (Euclidean, Aristotelian and Newtonian) models as having attained a kind of absolute certainty, but this does not mean, as is generally assumed, that he therefore rejects the possibility of other, equally valid models being developed.⁴ On the contrary, as we shall see, Kant's contribution was to show that these classical models can be regarded as absolutely valid only when certain limitations are placed on them, by viewing them from a particular perspective. Indeed, two sentences after referring to his use of geometers and physicists as exemplars, he adds that "pure speculative reason has this peculiarity, that it can measure its powers according to the different ways in which it chooses the objects of its thinking [i.e. its different perspectives on objects]" [K2:xxiii, emphasis added]. And this should serve at least as a hint that Kant intends his System to provide a philosophical basis for regarding geometry, logic, and physics as capable of different, but equally valid, non-transcendental formulations.

An even clearer hint comes in Kant's first published writing (1747), when he poses the problem: "The ground of the threefold dimension of space is still unknown" [K1:23]. He admits that the law which determines that space "has the property of threefold dimension...is arbitrary, and that God could have chosen another... law [from which] an extension with other properties and dimensions would have arisen. A science of all these possible kinds of space would undoubtedly be the highest enterprise which a finite understanding could undertake in the field of geometry" [24]. He suggests the law in question might be "that the strength of the

action [of substances] holds inversely as the square of the distances" [24]. However, he clearly recognizes that "the necessity of the threefold dimension" is not a logical necessity, "but rather...a certain other necessity which I am not as yet in a position to explain" [23]. His suggestions in this initial inquiry, he says, "may serve as an outline of an inquiry which I have in prospect" [25]. Thirty-four years later this "prospect" finally became a reality!

The necessity of the three-dimensional nature of space is explained in K2 as resulting from the fact that space is the pure form of our sensible intuition [K2:37-45]. Martin gives a detailed account of the mathematical implications of this theory. He explains that the intuitive character of mathematics means, for Kant, "that mathematics is limited to objects which can be constructed" [M1:23]. In other words, Kant's mature position is that intuition "limits the broader region of logical existence...to the narrower region of mathematical existence" [25]. Nevertheless, Kant did not give up his early insight regarding the logical possibility of non-Euclidean geometries: "there can be no doubt that it was clear to Kant that in geometry the field of what is logically possible extends far beyond that of Euclidean geometry" [23]. Indeed, he claims that "under the Kantian presuppositions it is not only possible but necessary to assume the existence of non-Euclidean geometries" [18]. Martin adds: "Non-Euclidean geometries are logically possible but they cannot be constructed; hence they have no [real] mathematical existence for Kant and are mere figments of thought."⁵

What Martin and other commentators fail to recognize is that, just because they are "figments of thought" does not mean that a Kantian must immediately reject non-Euclidean geometries as useless speculation. His main point is not that they are physically impossible, but that it is impossible to form an image or picture of what they would look like. Thus, after arguing in K1 that multi-dimensional physical spaces are logically possible, Kant adds the important qualification that "in anything representable through the imagination in spatial terms, the fourth [spatial dimension] is an impossibility" [K1:23]. He then connects them directly to the Leibnizian conception of many possible "worlds", which, he argues, can be regarded as not just possible, but "probable", only on the assumption "that those many kinds of space, of which

I have just spoken, are likewise possible" [K1:25]. Kant is not here making a positive claim about the real existence of non-Euclidean geometries, but only suggesting that we keep an open mind (which, on the traditional interpretation, he himself did not do). Before we investigate in more detail this issue of the conceptual status of non-Euclidean geometries, let us examine the actual importance of Euclidean geometry to the overall validity of Kant's System.

In the Aesthetic of K2 Kant seems to argue at several points from the validity of Euclidean geometry to the transcendental ideality of space. He says in K2:41, for example, that "geometrical properties are one and all apodeictic, that is, they are bound up with the consciousness of their necessity; for instance, that space has only three dimensions."⁶ Thus, he argues, "the only explanation that makes intelligible the possibility of geometry" is that space is a formal condition of experience, imposed on objects by the human mind. Later, he hinges his argument again on a similar reference to the need to explain the apodeictic certainty of mathematical propositions, a type of certainty which "is not to be found in the a posteriori" [K2:57]. And in K2:64-66 he provides his most lengthy argument of this sort: he takes the certainty of geometrical propositions as the starting-point (presupposed on the basis of his arguments in K2:14-17), and argues that this requires us to regard space as a "pure a priori intuition". Although Euclid is never mentioned by name in the entire Critique, such passages clearly indicate that Kant believed Euclidean geometry had attained a kind of certainty and necessity which places it beyond question.

As a result, Strawson argues that in the Aesthetic "the doctrine of the transcendental subjectivity of space rests on no other discernible support that [sic] that provided by the argument from geometry"⁷ Kant himself, however, does not present the few brief arguments "from geometry" as proofs of his theory of space, but rather he presents his theory of space as a support for a proper explanation of the necessity of geometrical propositions. The difference really rests on a different conception of the importance of arguments in the philosophical task itself: analytic philosophers such as Strawson regard a good theory as one that is based on good arguments; synthetic philosophers such as Kant, by contrast, regard a good argument as one

which arises out of a good theory. In other words, Kant's arguments "from geometry" are not intended to serve as the basis for anything; rather, they are intended to demonstrate that a "Copernican" view of space as transcendently ideal--i.e., as a subjective, a priori form of experience--provides the basis for an explanation of the otherwise inexplicable necessity of geometrical propositions.

Moreover, Kant does not take an entirely uncritical attitude towards Euclidean geometry. He uses it in the above-mentioned ways not because he has no other way to defend his theory of the transcendental ideality of space, but because the consensus of opinion in his day was that Euclidean geometry was undeniably true; hence it could be taken as an unproblematic premise in a way which would be impossible today. However, as we shall see, that very change is due in large part to Kant. For even though he takes the certainty of geometrical propositions as the starting point for a few of his arguments, his conclusion denies the validity of the traditional belief that such propositions apply to a physical reality called "absolute space" [K2:54-56]. Therefore, rather than siding with the consensus of opinion on the relevance (or irrelevance) of Kant's theory of geometry for contemporary philosophy of science, we must now look more closely at the perspective from which he regards geometrical propositions to be necessarily true.

Is Kant assuming in the Aesthetic that Euclidean geometry provides the true explanation of the real structure of physical space? Let us look more closely at Kant's actual position. Kant concludes his "Transcendental Exposition of the Concept of Space" with a summary, which is intended to serve as a warning:

The transcendental concept of appearances in space...is a critical reminder that nothing intuited in space is a thing in itself, that space is not a form inhering in things in themselves..., and that what we call outer objects are nothing but mere representations of our sensibility, the form of which is space. The true correlate of sensibility, the thing in itself, is not known, and cannot be known, through these representations; and in experience no question is ever asked in regard to it. [K2:45, emphasis added]

Here Kant is clearly warning the reader not to regard the arguments of the Aesthetic, which

adopt the transcendental perspective, as applying also to the empirical perspective. For, as he puts it quite bluntly, the sorts of questions he asks in this part of the Critique would not even arise if we limited our attention to the empirical perspective. Yet this warning has been overlooked or ignored by most of Kant's critics, with the result that Kant's position in the Aesthetic is probably the most frequently rejected part of the entire Critical System.

One of the most unfortunate results of the tendency to ignore Kant's warning against neglecting the perspectival character of his arguments is that he is interpreted as saying that Euclidean geometry is necessarily true of the physical world. In fact, a careful reading of the Aesthetic reveals that he never says anything of the kind! Rather, his whole argument is intended to draw the reader away from such empirical questions and towards questions concerning what is "bound up with [human] consciousness", and is therefore "apodeictic" [K2:41] in a completely non-physical (or meta-physical) way. The Aesthetic can only be understood as presenting a coherent argument once we recognize that in it Kant is not doing physics! Rather, he expects us to join with him in limiting our attention to the transcendental perspective. Viewing it in this way enables us to see that Kant is using geometry as an example--a test case--and not as an essential element in his system. Kant himself makes this quite clear when he introduces his argument from geometry in K2:64-66 in the following way: "To make this certainty [i.e. the certainty of his view that space is an a priori form of intuition] completely convincing, we shall select a case by which the validity of the position adopted will be rendered obvious..." [K2:64-65]. Obviously, he does not think the transcendental ideality of space depends at all on the validity of Euclidean geometry, but rather, vice versa!

In what sense, then, does Kant believe Euclidean geometry can rightfully claim apodeictic certainty? Certainly not, as is often assumed, by pointing to the empirical world and saying "See, it's true!" That would be to remain in the empirical perspective, which, as he says in K2:A24, can only tell us about "the contingent character of perception". Rather, its necessity can be explained only from the transcendental perspective: only by regarding geometrical propositions as describing the way in which we must present space to ourselves in our sensible

experience. Kant is arguing that Euclidean geometry describes the form of our perception of things in space, not the way they are actually related. And this, as we shall see, is not only plausible even today, but it leaves open a place for other geometries which might adapt the Euclidean model in such a way that it can apply to empirical reality itself (i.e. to the physics of space).

Just what, then, is Kant's attitude towards the empirical applicability of Euclidean geometry, and how important is the Euclidean structure of physical space to the validity of Kant's theory? Moreover, what exactly does Kant mean by his doctrines of "outer sense" and "pure intuition"? Kant begins his explanation in K2:37 by saying: "By means of outer sense, a property of our mind, we represent to ourselves objects as outside us, and all without exception in space." This "metaphysical exposition" is not intended to describe the way we experience space or objects in space; that would be an empirical concern. Rather, it is intended to describe the nature of the human subject. Without straying into the difficult subject of the meaning of "pure intuition",⁸ we can point out that one of the main points Kant is making in the Aesthetic is that, viewed from the transcendental perspective, space and time are two distinct forms of sensibility inhering in the human subject, despite the fact that in experience we always find them together. (The significance of this point will be discussed in section III.) The real thrust of Kant's argument is simply to point out that transcendental reflection requires us to distinguish between these two sources of material for knowledge, and to recognize that each, viewed separately, has a fixed, predetermined form. But this form is transcendental and relates to appearances; it is valid, therefore, "only in us" [K2:59], and leaves open the question of whether or not exactly the same forms will hold true when objects are regarded from the empirical perspective as being "outside us" [as in K2:37; see also 275-276].

According to Kant, therefore, Euclid's geometrical system is a transcendental abstraction from actual experience, and only because of this fact--not because the world is "really" structured in this way--can his system claim certainty and necessity. Kant is claiming that if we abstract space and time separately from our experience of the real (empirical) world,

and consider the necessary requirements for perception, then the resulting picture of this abstract, spatial world (as opposed to the empirical world of space-time) will be Euclidean. Kant regards this as a kind of "brute fact" about how the human mind is structured [see e.g. K2:42], in much the same way as he views the categories [K2:145-146,150-151], the schemata [K2:180-181], and the moral law [see e.g. K3:31] as brute facts; but we have no way of knowing whether or not they hold true for all rational beings [see e.g. K2:71-72; K3:72]. This conclusion is considerably less problematic than the view traditionally attributed to Kant, that physical space is actually Euclidean, because most geometers and physicists would readily agree that the only world we can "picture" with our imagination, and so also the ordinary world as we see it, is Euclidean.⁹

If, then, we assume that Kant was correct in his assumption that our sensibility is limited to a Euclidean picture of the world, what are the implications of this theory for geometers themselves? Kant's view is that the Euclidean nature of space is actually a transcendental condition for the very possibility of our perception of space, so that spatial objects viewed as appearances must assume this form. But Kant clearly understands that this transcendental perspective is primarily of interest to the philosopher; if Kant had ever had the opportunity to share his lunch with an Einstein, he would have readily admitted that the scientist is fully justified in viewing the world empirically, so that physical objects can be treated as independent "things in themselves".¹⁰ This implies that the geometer (or physicist) who accepts Kant's transcendental perspective does not need to assume that the physical world itself conforms to the way we must view it. That is, Kant's theory, by clearly distinguishing between the transcendental and empirical perspectives, and by associating Euclidean geometry exclusively with the former, actually raises the question as to whether or not some alternative geometry, though not picturable to our sensibility, might conform more closely to the way objects are actually structured in (empirically) real space.

The chief objection against this interpretation of Kant's intentions is, no doubt, that it flies in the face of the almost universally accepted assumption that Kant is trying to guarantee

the applicability of Euclidean geometry to the physical world. Unfortunately, Kant himself never clearly states just what his intentions are in this respect. As a result, the way of reading Kant which was natural in the eighteenth century, when the question of the actual structure of the physical world would hardly be raised by most readers, because Euclidean geometry was generally accepted as unquestionably true, has become the traditional interpretation of Kant's own intentions, and has never been significantly challenged. Thus, for example, when Kant says in K2:121 that "the concepts of space and time", regarded as pure intuitions, "independently of all experience...make possible a synthetic knowledge of objects", the natural assumption is that "objects" here refers to real empirical objects. Yet this is not Kant's intention at all: the "objects" of which we have synthetic a priori knowledge by means of Euclidean geometry are exclusively appearances, and as such are viewed transcendently.¹¹

Strawson's analysis of Kant's theory of geometry provides a good example of the traditional interpretation. He explains that geometry can be viewed as relating either to a set of phenomenal (and thus unfalsifiable), pictures, or to the logic behind them, or to their application to objects in physical space. He states "that Kant's theory of pure intuition can be construed as a reasonable account of the nature of geometry in its phenomenal interpretation" [S1:284]. He then asserts that Kant intended "to use his insight into the necessities of phenomenal geometry to resolve... the difficulty created by the apparently necessary application of Euclidean geometry to physical space" [284-285]. Thus, he continues, Kant's fundamental error...lay in not distinguishing between Euclidean geometry in its phenomenal interpretation and Euclidean geometry in its physical interpretations... Because he did not make this distinction, he supposed that the necessity which truly belongs to Euclidean geometry in its phenomenal interpretation also belongs to it in its physical interpretation. He thought that the geometry of physical space had to be identical with the geometry of phenomenal space. [285]

Significantly, Strawson does not refer to a single text to support his acceptance of this traditional interpretation--i.e. his assumption that Kant's notion of pure intuition corresponds to

phenomenal geometry and that its empirical application entails an identification of physical geometry with phenomenal geometry. By contrast, if we regard transcendental geometry in the way I have suggested, then Kant's doctrine of pure intuition "accounts" for phenomenal geometry only in the sense that it serves as its transcendental foundation. That is, the empirical application of pure intuition gives rise to phenomenal geometry, but is never directly related by Kant to physical space. Thus, Strawson's "fundamental error" lay in not distinguishing between Euclidean geometry as transcendental and Euclidean geometry as phenomenally applied. Kant's silence on the subject of physical geometry need not be interpreted as an identification of phenomenal with physical, but may simply reflect his recognition that physical geometry (as a posteriori) is a subject which need not be addressed by the transcendental philosopher.

Strawson does quote K2:155n to make quite a different point about the construction of phenomenal figures [S1:289]. Yet this text actually denies the very (traditional) assumption which Strawson takes for granted! In K2:155n Kant says:

Motion of an object in space does not belong to a pure science, and consequently not to geometry. For the fact that something is movable cannot be known a priori, but only through experience. Motion, however, considered as the describing of a space, is a pure act of the successive synthesis of the manifold in outer intuition in general by means of the productive imagination, and belongs not only to geometry, but even to transcendental philosophy.

Kant's main point (which Strawson ignores) is that "pure [i.e. a priori] science" must be carefully distinguished from applied, or empirical science, so that once "facts" such as some particular motion are taken into consideration, we are no longer even talking about geometry (whether Euclidean or non-Euclidean).¹² In other words, Kant seems to be saying in this footnote that the applicability of geometry to objects in physical space is a problem which lies outside the bounds of geometry (and transcendental philosophy)! And this implies, of course, that Euclidean (or any other pure) geometry does not necessarily hold true of physical objects when they are viewed as objects of empirical science.

This interpretation of K2:155n makes sense only if we remember that Kant defines

geometry as "a science which determines the properties of space synthetically, and yet a priori" [K2:40; see also 120]. Given this definition, of course, Kant could not admit any non-Euclidean geometries, because both of the types of geometry which Russell describes (viz. logical and physical [see note 11]) would not count as "geometry" in Kant's special sense. Only Euclidean geometry is synthetic and a priori, so only it is relevant to the transcendental philosopher. When this is understood, the question of whether or not physical space is Euclidean can be seen in its proper perspective, as a side issue relating not to the validity of transcendental philosophy, but only to the question of the significance of Euclidean geometry for empirical science.

III. The Empirical Application of Transcendental Geometry

Does this mean that the traditional interpretation of Kant, according to which he is attempting to give a transcendental guarantee that Euclidean geometry must be true of the physical world, is a groundless myth? Certainly not, for there are many passages in which Kant is indeed saying something along these lines. A prime example comes in the section entitled (significantly) "Axioms of Intuition", where Kant says:

This transcendental principle of the mathematics of appearances [i.e. that "All intuitions are extensive magnitudes" [K2:202]] greatly enlarges our a priori knowledge. For it alone can make pure mathematics...applicable to objects of experience. Without this principle, such application would not be thus self-evident; and there has indeed been much confusion of thought in regard to it. Appearances are not things in themselves. Empirical intuition is possible only by means of the pure intuition of space and of time. What geometry asserts of pure intuition is therefore undeniably valid of empirical intuition. The idle objections, that objects of the senses may not conform to such rules of construction in space as that of the infinite divisibility of lines or angles, must be given up... [206]

This text seems to lend unequivocal support to the traditional interpretation: here Kant clearly states his conviction that the transcendental validity of Euclidean geometry implies a

corresponding empirical validity.

The question is, just what does Kant mean here by "undeniably valid of empirical intuition"? Is there any way to accept Kant's view of the determining influence of the transcendental on the empirical, and yet to preserve some degree of autonomy for the empirical scientist (an autonomy which in other passages Kant seems to uphold)? Such questions did not occur to Kant, because the science of his day had not yet fully claimed its autonomy (from philosophy) as a thoroughly empirical discipline; hence Euclid's "transcendental" geometry was still believed by nearly all scientists and philosophers to be the last word on empirical matters as well. Kant was clearly influenced by his tradition in a way for which he can hardly be blamed. The important point, however, is that the perspectival character of his System points us directly to a way of supplementing his own explicit views in order to account for the modern developments. In this section I will attempt to demonstrate just how this can be done.

Kant in all likelihood believed Euclidean geometry to give a true account of the structure of the physical world. Nevertheless, even if we would now prefer a non-Euclidean account, the essential structure of his theoretical system can remain in tact, once we realize that the form of our sensible perception of objects in space, when all time-considerations are abstracted, is indeed necessarily Euclidean. And in this phenomenal sense, as a description of how we perceive the world, Euclid's system is regarded as correct even today. How, then, can a physical theory of curved space, with its corresponding non-Euclidean geometry, be compatible with the phenomenal validity of Euclidean geometry? Kant provides us with a way of solving this difficult problem.

When Kant says in K2:206 that the geometry of pure intuition is "undeniably valid" for empirical intuition as well, he is not denying that the physicist can conceive of a non-Euclidean geometry which actually holds true for aspects of space which human beings cannot perceive. On the contrary, the objects of "empirical intuition", which he is claiming must necessarily conform to Euclidean geometry, are objects of "possible experience", by which Kant means "objects which are within the range of human perception". And the fact which is too often

neglected in most accounts of Kant's supposedly miserable failure to foretell the future by providing for modern scientific advances is that these advances all have to do with "viewing" physical objects in their extreme manifestations--i.e. quantities which are either too small for human beings to perceive (as, e.g., in quantum physics) or too large for human beings to perceive (as, e.g., in the application of relativity theory to astrophysics). Yet, giving proper emphasis to this fact enables us to locate the real problem in Kant's view, which is simply that he failed to acknowledge that the objects of empirical science are not limited to perceivable objects. In other words, Kant's analysis of the limits of human knowledge neglected to consider the status of empirical sciences which experiment with quantities too small or too large ever to be intuited by the human sensibility. For example, the modern theory of physical space as "curved" is based on conceptions and calculations, not on perceptions of a space which actually appears curved. By simply recognizing that physical science does not always require direct intuition of the objects on which it experiments, we can therefore reconcile the modern views concerning the geometry of physical space with Kant's view of the transcendental necessity of our perception of Euclidean space.

Moreover, in the geometry of curved space, the perspective-lessness of the observer (or the unobservability of the perspective) is of utmost importance. For in order to perceive our space as curved, we would have to be able to observe it from the standpoint of some other space (outside of our own space) which is not curved. And this is precisely Kant's point about what man cannot do: his doctrine of pure intuition is intended to drive home the fact that it is "solely from the human standpoint that we can speak of space" [K2:42]. The equivalent requirement in Euclidean geometry can be accomplished, however, by adopting, as it were, the standpoint of time and looking at space from this perspective as nontemporal.¹³ This, in Kantian terms, is the crucial difference between Euclidean and non-Euclidean conceptions of space: there is no human standpoint from which the latter can be perceived.

Thus, as we have already seen, Euclidean geometry is a (transcendental) abstraction from experience: it arises only when the subject, his motion and his time, are removed from

observation (and in this sense, adopted as the standpoint). "Abstracting time" from our consideration of experience really means, therefore, regarding space as an object of spatial appearance--an intuited "picture" of the real world. In other words, Kant's theory is telling us that the human perceiver breaks experience up into temporal and spatial components: the reality we experience exists as a unified, space-time continuum; but the perceiver determines it in such a way that it appears to be Euclidean. Thus, the empirical object viewed from the transcendental perspective as in us (i.e. as an appearance) is Euclidean, even though that same object viewed from the empirical perspective as outside us (i.e. as a phenomenon in space-time) might be non-Euclidean.

When the scientist intentionally refuses to submit to the natural human tendency to perceive time and space as separate (i.e. refuses to let the inner-outer distinction influence his science), then the problem of determining the structure of space is immediately transformed from a static, transcendental inquiry, into a dynamic, empirical inquiry. (As a result, space is, in a sense, dehumanized.) Any resulting non-Euclidean theory of space is bound to be "counter-intuitive" (i.e. not picturable) for precisely the reasons Kant gives in his defence of the transcendental character of Euclidean space: since Euclidean geometry is, as Kant argued, the form of our intuition of space, other geometries can never be perceived, but only conceived, even if we subsequently discover that a non-Euclidean explanation, though purely conceptual, nevertheless enables us to explain certain phenomena better than a Euclidean one. This is because the other theories can be drawn out from experience, but are not read into experience by the human subject. For this very reason (i.e. by definition) only Euclidean geometry can be both synthetic (true of our abstract, nontemporal representation of our experience) and a priori (deriving its truth from a source other than experience); the synthetic applicability of all other geometries can be known only a posteriori (after collecting data based on cumulative experiments). In other words, our knowledge of the applicability of any non-Euclidean geometry will be contingent because it presupposes experience, whereas that of Euclidean geometry is necessary because it prefigures experience. And this is why Kant claims that no

non-Euclidean geometry can exist (i.e. can have real possibility): because it would have to be a posteriori, and therefore could not be pure (so it would not be a true "geometry" according to his definition).

What is almost always ignored in discussions of Kant's theory of geometry is that Kant himself makes some (very "modern") suggestions in the "Transcendental Dialectic" of K2 concerning the usefulness of just such non-intuitive theories for science. A theory which is conceptual but which contains no intuitable content cannot, he says, be regarded as establishing "empirical knowledge", but viewed as an "idea" it can serve as an hypothesis with definite heuristic value. He defines an "idea" as "a concept formed from notions and transcending the possibility of experience" [K2:377]. The "transcendental ideas", he explains, "determine according to principles how understanding is to be employed in dealing with experience in its totality" [378]. Kant's main effort in the Dialectic is, of course, to expose the fallacies which arise whenever such ideas are thought to constitute empirical knowledge. As a result, his lengthy appendix on "The Regulative Employment of the Ideas of Pure Reason" [670-698] is often ignored. In it he explains that ideas of reason can have a legitimate "immanent" use if we treat them "as if" they are true: "The hypothetical employment of reason is regulative only [not constitutive of knowledge]; its sole aim is, so far as may be possible, to bring unity into the body of our detailed knowledge" [675]. If Kant were alive today, it seems likely that he would apply this theory of ideas to any attempt to describe physical space as non-Euclidean. In other words, he would say that if a non-Euclidean concept of space enables scientists "to bring unity into the body of [their] detailed knowledge", then they are more than welcome (indeed, encouraged) to make such a conjecture, provided they never claim to have established empirical knowledge of its certain truth. And this seems to be quite an accurate account of the way modern scientists do view such theories!

Returning now to the quote from K2:206, we can see that Kant is not, in fact, contradicting the perspectival interpretation which I have been defending. For his claim is that pure intuition determines the nature of empirical intuition; any consideration of empirical

intuition assumes the transcendental perspective (that of the Aesthetic, where space and time are separately abstracted from experience), not the empirical perspective (that of the Analytic of Principles, where space and time are put back into experience, and where intuitions and concepts are synthesized by the imagination). And, as we have seen, Kant does not limit scientists to the empirical perspective, but encourages them to adopt the hypothetical perspective (that of the Dialectic, where concepts without intuitions are viewed as regulative ideas).¹⁴ So scientists who do not purport to be examining "objects of the senses" are not bound by Kant's theory to force the results of their inquiries into a Euclidean (sensible) mold.

IV. Concluding Remarks on Kant's Copernican Perspective

The novelty of this conclusion may be surprising at first; yet upon reflection it should be viewed as a natural implication of Kant's whole Copernican revolution. For his assumption that the subject determines (transcendentally) the perceived character of empirical objects always acts as a two-edged sword: he uses one edge to cut the traditional positions off from the domain to which they were formerly believed (erroneously) to apply; and with the other edge he protects those same positions from further attack by putting them in their proper place. Kant makes his dual motivations quite clear with respect to metaphysics, morality and religion in K2:xxx-xxxi. And there is no reason to suppose the implications of his Copernican perspective on geometry to be an exception. In fact, this Copernican perspective can be seen working in a remarkably similar way by comparing Kant's attitude towards Euclidean geometry with his attitude towards Aristotelian logic and Newtonian physics.¹⁵ However, such comparisons are beyond the scope of this paper.

Even in his earliest essay Kant shows an awareness of the perspectival distinction we have been examining: "body as mathematically conceived is a thing quite distinct from body as it exists in nature; and statements can be true of the former which cannot be extended to the latter" [K1:140]. And as we have seen, Kant's mature philosophy also leaves open a place for admitting a valid scope of application for non-Euclidean geometries, systems which do not

impose predetermined forms onto perceived objects, but which ignore the empirically perceived forms and view the objects "as if" they participate in a single, space-time reality. This may be precisely what Kant had in mind when he said the empirical scientist is permitted to ignore the implications of the transcendental perspective by treating objects as things in themselves [see P3:136].

Kant's first Critique is often regarded as a book on the philosophy of science--in particular, a book devoted to working out the epistemological underpinnings of Euclidean geometry and Newtonian physics, in order to demonstrate their absolute validity once and for all. I have argued in this paper, however, that this is a gross oversimplification of his actual intentions. Kant develops a "natural philosophy", a philosophy devoted to working out the epistemological underpinnings of the ordinary man's view of the natural world [K2:858-859]. To the extent that science abandons this ordinary standpoint it passes beyond the bounds of empirical knowledge and into the realm of hypothesis. Kant consistently views his task as that of constructing a "propaedeutic to the system of pure reason" [K2:xlili,25,869]. A "propaedeutic", he explains, is often "obtained last of all, when the particular science under question has been already brought to such completion that it requires only a few finishing touches to correct and perfect it" [K2:76]. In this sense, his system is a kind of philosophy of science on behalf of Euclid and Newton. (Indeed, it is no accident that the title given to the main part of the Critique, "Transcendental Doctrine of Elements", mimics the title of Euclid's classic book.¹⁶) However, a "propaedeutic" is also forward-looking: in Kant's case it points to the "metaphysics" of the future [K2:878]. (Thus, the title of the companion volume to K2 is *Prolegomena to Any Future Metaphysics...*). Moreover, as we have seen, by demonstrating that the classical scientific systems depend on and arise out of the employment of clearly defined transcendental limits, Kant's system presents science with a series of open questions as to how far other, hypothetical models can be developed and applied to the world we experience.

Since 1781 there have been four major revolutions which are of particular interest to philosophers of science: the development of non-Euclidean geometries by Lobachevsky in

1840 and Riemann in 1854, together with their subsequent application as explanations of the structure of real space; the foundations set by Frege in 1879 for the development of sophisticated, extra-Aristotelian logical systems; the innovation of relativity physics by Einstein in 1905; and the elaboration of quantum mechanics by Bohr and Heisenberg in 1927. For too long these surprising revolutions have been regarded as evidence against a Kantian interpretation of the world. Yet if the foregoing interpretation of Kant is correct, then it is high time we come to realize--especially in light of the current trends in the philosophy of science, the trends away from any attempt to give definitive statements as to what science is, and towards a more skeptical, historical/methodological approach--that the world of modern science is a thoroughly Kantian world, and that this is no coincidence! The time has come, that is, to stop asking whether Kant's philosophy can still be valid in light of modern developments, and to begin asking instead, whether these developments would have arisen had Kant not explicated the world view on which they have their philosophical foundation. My intention in this article has been to demonstrate that, in fact, the first of these revolutions is consistent with, if not based upon, the very Copernican turn which Kant's System established. Further investigation would reveal that in each case these new developments are based on the same principle, the principle which Kant used to structure the fabric of his entire System, the principle of perspective.

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FOOTNOTES

1. All references will be included in the text using the abbreviations given in the Bibliography. References to K2 (Kant's first Critique) will follow the second edition, except where the material is unique to the first edition, in which case the page number will be prefixed with "A". References to Kant's other works cite the pagination of the Berlin Academy edition of Kant's works.

2. S2:83. Churchland makes a similar claim in C1:84: "Both Euclidean Geometry and Newtonian physics have since [Kant's day] turned out to be empirically false, which certainly undermines the specifics of Kant's story." According to Paton, "The possibility of new mathematical concepts is certainly not excluded by Kant's theory" [P11:161]; however, in his opinion, "Kant's doctrine [of space and time] is altogether too simple in the light of modern discoveries" [163]. Nevertheless, he thinks we can at least admit that Kant's theory of the synthetic a priori nature of geometrical propositions is consistent with Euclid's methods [159]. Ewing is more optimistic. He says in E2:47 that Kant's view of Euclidean geometry "must not be dismissed at once as necessarily undeserving of any further consideration", for even "Einstein himself admits that space may be Euclidean" [46]!

Fortunately, some Kant-scholars do recognize that the overthrow of classical scientific paradigms does not require the rejection of Kant's Critical System. This "old-fashioned charge", as Buchdahl puts it, "is wide of the mark" [B1:146]. At worst, he adds, the modern revolutions indicate only that the Critical System's "'application' can no longer take place in the way envisaged by Kant, and that the material to which it was 'applied' stands in need of conceptual revision" [156]. He maintains that Kant intended his System to "function as a series of constraints on the choice of possible hypotheses and of possible theoretical formulations" [146], of which Newtonian physics is but one type. This view of Kant as proposing a highly "progressive" philosophy of science is supported by Fang [F1:106], who explicitly states (but

unfortunately, leaves undeveloped) that K2 is "perfectly compatible with the philosophy of Relativity Theory and Quantum Mechanics, to say nothing of modern mathematics" [123]. Ellington defends the same claim in a bit more detail in E1:xxx.

3. The foregoing perspectival interpretation is used in P4 to explicate the formal structure of Kant's System. The notion of a perspective is defined, and its application to Kant's four main perspectives is demonstrated, in P7. The perspectival way of interpreting Kant is applied to his theory of the object of knowledge (e.g. terms such as "thing in itself" and "appearance") in P3. And it is applied to his moral philosophy in P5 (from which is taken much of the material in the current paragraph in the main text). In several other articles I have used this way of interpreting Kant to answer criticisms of Kant made by Michael Polanyi [in P6], Philip Kitcher [in P9] and Saul Kripke [in P10]. The justification of the entire Critical enterprise, with its questionable assumption of the "thing in itself", is defended on perspectival grounds in P1 and P2. Finally, I have discussed Kant's general attitude towards science (or "natural philosophy") in P8.

The summary of my interpretive framework given here differs in two respects from my previous publications. First, I now refer to the third standpoint (previously the "empirical") as "judicial" in order to avoid confusing it with the empirical perspective within each standpoint (a potential confusion Kant himself recognizes in K4:178-179). Although this standpoint is "empirical" in the sense that it does deal with particular aspects of man's experience (e.g. aesthetic or religious aspects) much more fully than in reasoning based on the theoretical or practical standpoints, this use of "empirical" must be carefully distinguished from the use in the important transcendental-empirical distinction. Moreover, in light of K2:739 ("There is no need of a critique of reason in its empirical employment") the label "judicial" (i.e., relative to judgment) more adequately reflects the transcendental status of this standpoint, as well as the fact that its scope is broader than the empirical perspective within each system.

The second change is that I now refer to the fourth (previously "practical") perspective in each of Kant's three systems as the "hypothetical" perspective. This avoids the potential

confusion between it and the practical standpoint, as adopted in K2. My use of "practical" was potentially misleading because Kant normally equates "practical" with "moral", whereas the fourth perspective of the theoretical and judicial standpoints is not limited to morality. The word "hypothetical" is an appropriate replacement because it suggests the "as if" character of all conclusions established from this perspective. (This is made especially obvious in the Dialectic of K2.)

4. Thus Gottfried Martin suggests in M1:96 that a helpful way of describing Kant's "a priori" approach to science is to see it as a way of "considering Aristotelian logic as the foundation of all logics, Euclidean space as the foundation of all other spaces, and classical physics as the foundation of all other kinds of physics." Unfortunately, Martin does not fully develop the implications of this suggestion. It describes quite succinctly, however, the interpretation of Kant's intentions which I am defending in this paper, according to which a priori science differs in kind from a posteriori sciences inasmuch as the latter depends on adopting a transcendental rather than an empirical perspective.

5. M1:24. Martin explains that the search for non-Euclidean geometrical systems was well under way by Kant's day [M1:17-18]. In fact, Kant's friend Lambert practised an early form of non-Euclidean geometry, so it is likely that Kant was well aware of these developments.

Although Friedman (rightly) thinks Kant believes "there is no way to draw, and thus no way to represent, a non-Euclidean straight line" [see note 9 below], he (wrongly) concludes "the very idea of a non-Euclidean geometry is quite impossible" for Kant [F2:488]. He never says what he means by "idea", yet he uses the word in similar ways several times: "pure intuition [for Kant] cannot be said to provide a model for Euclidean geometry at all; rather it provides the one and only possibility for a rigorous and rational idea of space" [505; see also 504]. Unless Friedman has a very unusual (and unKantian) meaning for "idea", such claims are quite obviously unfounded. Indeed, I will argue in section IV that Kant's theory of "ideas" in the

Dialectic is precisely the proper place to locate the possibility of non-Euclidean geometries in a modern reconstruction of Kant's System.

In F2:502-503 Friedman claims Kant "has no notion of possibility on which both Euclidean and non-Euclidean geometries are possible." To defend his view he refers to various texts in the Postulates [K2:265-294], where Kant is intentionally limiting his attention to real possibility. Friedman describes Kant's position with admirable clarity when he mentions "two notions of possibility: 'logical possibility,' given by the conditions of thought alone; and 'real possibility,' given by the conditions of thought plus intuition" [F2:503]. However, he then makes the outrageous claim: "this line of thought employs a notion of logical possibility that is completely foreign to Kant." In the Dialectic, and throughout Kant's Critical System, Kant's distinction between logical and real possibility plays an essential role in his arguments [see e.g. K2:xxviii]. (This point is forcefully argued by Allison in A1.) Friedman's mistake is to limit his understanding of Kant's view of possibility to the strict comments regarding the limitations of real possibility which he makes in the Postulates. (In any case, such "real" possibility refers not to what is physically possible, but to what can be constructed or represented by the human perceiver.) That Kant believes non-Euclidean geometries are logically possible is made unambiguously clear in K2:268: "there is no contradiction in the concept of a figure which is enclosed within two straight lines". A better way of expressing the point Friedman is trying to make, therefore, would be to say Kant downplays the importance of merely logical possibility.

6. "The apodeictic certainty of all geometrical propositions" is also exemplified in K2:A24 by referring to the Euclidean doctrine "that there should be only one straight line between two points".

7. S1:277. Martin demonstrates, by contrast, that in K2:38 alone there are two carefully structured arguments for the transcendental ideality of space which do not refer at all to geometry: indeed, he traces the significance of these arguments back to Plato and Aristotle

[M1:29f,32f]. Moreover, Pippin not only argues explicitly against Strawson's claim, but provides a list of other recent works which do the same [P12:55n; see also 73-74n]. Strawson's interpretation is criticized most thoroughly in H3.

8. I discuss the proper interpretation of Kant's doctrine of "pure intuition" in P9 [see also note 11 below]. The typical misinterpretation is made by Melnick in M2:51-54 when he refers to pure intuition as "an imaginative act or sequence" by which something is constructed. As long as "imagination" is understood in its ordinary sense (as a kind of internal perception) rather than in Kant's special, transcendental sense (as a kind of pre-conscious requirement for perception), such a description of Kant's theory of pure intuitive construction is sure to be misleading. In K2:299 Kant clearly explains that, because the basic principles of Euclidean geometry are pure intuitions, they are essentially non-sensible ("generated in the mind completely a priori"); only if the mathematician can construct them in empirical intuition, as "an appearance present to the senses", can we actually experience them, even though they are originally "produced a priori" in the transcendental recesses of our mind.

9. There is some potential for misunderstanding on this matter of the picturability of non-Euclidean geometrical figures. Hawking defends the established position in H1:24: "It is impossible to imagine a four-dimensional space." Nevertheless, Martin argues "that there are ways in which non-Euclidean geometries can also be constructed, by purely analytical means or by constructing Euclidean models of non-Euclidean geometries" [M1:25]. Such "construction", however, is logical in the former case, and Euclidean in the latter, so neither type provides actual non-Euclidean pictures. Indeed, it seems clear enough that, whenever we make pictures of higher-dimensional spaces, we always do so by adding specially defined lines onto Euclidean pictures. Even simple non-Euclidean principles, such as the assertion that two different straight lines can pass through two points, can be pictured only in a Euclidean way, such as by drawing at least one of the lines as curved, and pretending it is straight. Thus,

Reichenbach, despite his thorough discussion of the picturability of non-Euclidean geometries in R1:37-92, agrees that a non-Euclidean geometrical proposition "contradicts the human power of visualization" [3].

10. See my defence of this claim in P3:136.

11. See P3:130-134. In opposition to Kant, Russell argues that there are only two kinds of geometry now thought to be valid: "pure geometry" and "geometry as a branch of physics... Thus of the two kinds of geometry one is a priori but not synthetic, while the other is synthetic but not a priori" [R2:743]. This view, which entails that the Euclidean use of pictures is obsolete and can be--indeed, ought to be--replaced by purely analytic, logical representations, is widely accepted today [see e.g. C2:9-11 and F2:455-457]. Thus in R3:145 Russell chides Kant for inventing "a theory of mathematical reasoning according to which the inference is never strictly logical, but always requires...'intuition'. The whole trend of modern mathematics...has been against this Kantian theory."

Strawson calls Russell's twofold distinction "the positivist view" [S1:278f] and argues that "a third way" is to restrict geometry to an analysis of the way things appear [286]. This way of viewing Euclidean geometry is the only respect in which its propositions can be called self-evident, and is important especially "in the initial stages of learning geometry" [286]. Moreover, Strawson rightly sees that, as we shall see in section III, this "phenomenal geometry" plays an important role in the empirical application of Kant's theory [287]. The main danger in Strawson's view is a tendency to ignore Kant's distinction between a picturable "image" and a rational "schema": the latter "can exist nowhere but in thought. It is a rule of synthesis of the imagination, in respect to pure figures in space" [K2:180]. So the picturable (Euclidean) image becomes applicable only through such a schema, which is in itself not a visual representation. Thus, what Strawson calls "phenomenal geometry" is the empirical effect of the transcendental operations of pure intuition. As such, the visual images themselves should be regarded as

epiphenomena of Kant's theory.

Friedman contrasts Russell's view with what he calls the "anti-Russellian interpretation", which views Kant as agreeing with Russell that mathematical reasoning is analytic, but as viewing the axioms on which such reasoning is based as synthetic [F2:486-489; see also 502]. After examining Kant's views on the empirical procedure of mathematical proof, which he mistakes for the entirety of Kant's view, Friedman confidently concludes that this alternative is "rather obviously untenable and definitely unKantian" [498]. The reason, he explains, is that arithmetic, for Kant, "differs from geometry precisely in having no axioms" [490]. "For Kant...arithmetical propositions are established by calculation, a procedure that is sharply distinguished from logical argument in being essentially temporal" [491]. This "distinction between calculation and logical argument", he suggests, "is perhaps most basic to Kant's conception of the role of intuition in mathematics" [492].

Friedman's position is seriously inadequate, however, in several respects. First, aside from a brief comment in a footnote [F2:49n], he never defends his position against the opposite view, defended at length by Martin, "that Kant discovered the axioms of arithmetic...and that the axiomatic theory of arithmetic starts from these Kantian discoveries" [M1:18]. Secondly, his belief that the synthetic character of mathematics is related for Kant to the temporal character of calculation reveals quite a naive interpretation of "pure intuition" (much like that of Kitcher, which I thoroughly criticize in P9). Thus he opines, "the whole point of pure intuition [for Kant] is to enable us to avoid rules...by actually constructing the desired instances" [469]. Such geometrical construction (like all arithmetical calculation, or for that matter, all logical argumentation), however, is thoroughly an a posteriori activity, and misses the whole point of Kant's emphasis on the a priori [see P10, esp. section IV]. Friedman quotes K2:xi-xii in defence of his conclusion about the reasoning process [F2:501]. Yet he reads quite an unKantian meaning into the word "method". "The true method" for demonstrating "the properties of the isosceles triangle", Kant explains, is "not to inspect what he discerned either in the figure [empirically, i.e. synthetic a posteriori], or in the bare concept of it [logically, i.e.

analytic a priori]...; but to bring out what was necessarily implied in the concepts that he had himself formed a priori..." [K2:xi-xii]. Friedman thinks "method" here means "step-by-step procedure". Yet Kant explains this "true method" by referring directly to a "true perspective", or way of viewing the (empirical) procedures involved in constructing such proofs. Thus there are grounds for serious doubt concerning Friedman's confident conclusion: "I do not see how there can be any doubt...that Kant's 'true method' of geometry is precisely Euclid's procedure of construction with straight-edge and compass" [F2:501]. On the contrary, nothing could be further from Kant's mind! Kant is not suggesting that the philosopher should measure metaphysics with a straight edge and compass (i.e. a posteriori)! Rather, he is arguing that the philosopher, like the pure geometer, should adopt a transcendental (synthetic a priori) perspective. Unfortunately, Friedman (like Russell) seems to know a lot more about geometry than he does about the purposes and methods of Kant's Critical System.

12. Friedman admits that K2:155n implies a distinction between pure and applied mathematics [F2:482n], yet he asserts with little actual explanation or argument [see note 15 below] that Kant had nothing corresponding to "our modern distinction between pure and applied mathematics" [F2:504n]. My interpretation of K2:155n implies that Friedman's view is quite mistaken. Kant's most important perspectival distinction--between the transcendental and empirical perspectives--can be regarded as a forerunner of this modern distinction. The differences which disturb Friedman should not be described in terms of "modern" versus "out-of-date", but rather in terms of "mathematical" versus "philosophical". Moreover, Martin explains, in defending the traditional interpretation, that Kant's goal was to solve the problem of "why Euclidean geometry, in spite of being a pure construction in thought, is valid for Newtonian physics" [M1:36]. "By discovering the connection between mathematics and natural science Kant fundamentally went far beyond the [classical] concept of applied mathematics. What we have is not a ready-made mathematics which is applied in physics, but one fundamental human faculty which is active in mathematics and in physics" [36]. And this is

much closer to the modern ways of viewing this problem than Friedman acknowledges.

13. This standpoint is reflected in an interesting way in Heidegger's interpretation, when he says "the ego cannot be conceived [by Kant] as temporal, i.e. as intra-temporal, precisely because the self originally and in its innermost essence is time itself" [H2:200-201].

The close correlation between space and time as forms of human intuition suggests that there ought to be a synthetic a priori science of time, corresponding to transcendental (Euclidean) geometry. Paton cites the lack any clear reference to such a corresponding science in the Aesthetic of K2 as a weakness of Kant's exposition [P11:128]. Wolff argues in W1:230 that mechanics is to time as geometry is to space. Most interpreters, however, assume arithmetic to be the science in question, since Kant uses an example from arithmetic in K2:15-16 just before introducing his example from geometry, both of which are meant to clarify the nature of synthetic a priori judgments. (He also pairs together the "propositions of arithmetic or geometry" in K2:764.) Friedman even claims that arithmetic is the primary science for Kant, since inner sense is prior to outer sense [F2:495-496]. But this question of priority is debatable, since Kant does discuss space and outer sense before time and inner sense in the Aesthetic, and because space is given a measure of priority over time in the Refutation of Idealism [K2:274-279], where Kant argues that our consciousness in time presupposes something permanent in space. My view is that from the transcendental perspective, time has priority, whereas from the empirical perspective, space has priority.

14. This interpretation of the correlation between the sections of K2 and the perspectives Kant adopts is defended in P4 and P7, and in more detail in my forthcoming book, *Kant's System of Perspectives*, chs 2-4,7.

15. The only redeeming aspect of Friedman's otherwise abstruse treatment of Kant's theory of geometry [see notes 5, 11, 12] is that he correctly stresses the close relationship between Kant's

attitudes towards Euclidean geometry and Aristotelian logic [F2:456]. Unfortunately, after pointing out that "our logic, unlike Kant's is polyadic [quantifier-dependent] rather than monadic (syllogistic)" [460], he infers that Kant was simply wrong, though he adds that we should not blame him too harshly. His argument runs something like this: (1) our modern view of logic and mathematics began with Frege in 1879; (2) Kant developed his Critical philosophy a century before Frege's revolution; therefore (3) Kant cannot have suggested or had in mind anything like our modern distinctions [see e.g. F2:481-483,488-489,500n,503n]. In a sequel to the present article I plan to argue, by contrast, that Kant laid the groundwork for, and in some respects anticipated, the modern innovations to which Friedman refers [see note 16 below]. The reason this is not often recognized is that the foundations he laid were thoroughly philosophical, not mathematical.

16. Moreover, just as Kant's "Elements" begins with an exposition on how to view Euclid's Elements in its proper perspective, so also it continues by doing the same with three other classical traditions. The "Analytic of Concepts" develops a doctrine of categories which enables us to view Aristotle's classic book, Categories, in its proper perspective. The "Analytic of Principles" develops a doctrine of principles which enables us to view Newton's classic book, Principia, in its proper perspective. And the "Dialectic" develops a doctrine of ideas which enables us to view the corresponding classical theory of Plato (as expressed in his Dialogues) in its proper perspective. The first Critique is an attempt to show that each of these classical theories has a proper role to play in a complete system of theoretical philosophy, but that the validity of each can be established only by limiting each to a specific perspective. Without a clear recognition of Kant's Critical attitude towards his tradition (an attitude which always includes both negative and positive aspects), the significance of this modern classic cannot be fully understood.