Authentication of moving top-k spatial keyword queries

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Authentication of Moving Top-k Spatial Keyword Queries

Dingming Wu, Byron Choi, Jianliang Xu, and Christian S. Jensen

Abstract—A moving top-k spatial keyword (MkSK) query, which takes into account a continuously moving query location, enables a mobile client to be continuously aware of the top-k spatial web objects that best match a query with respect to location and text relevance. The increasing mobile use of the web and the proliferation of geo-positioning render it of interest to consider a scenario where spatial keyword search is outsourced to a separate service provider capable at handling the voluminous spatial web objects available from various sources. A key challenge is that the service provider may return inaccurate or incorrect query results (intentionally or not), e.g., due to cost considerations or invasion of hackers. Therefore, it is attractive to be able to authenticate the query results at the client side. Existing authentication techniques are either inefficient or inapplicable for the kind of query we consider. We propose new authentication data structures, the MIR-tree and MIR+ tree, that enable the authentication of MkSK queries at low computation and communication costs. We design a verification object for authenticating MkSK queries, and we provide algorithms for constructing verification objects and using these for verifying query results. A thorough experimental study on real data shows that the proposed techniques are capable of outperforming two baseline algorithms by orders of magnitude.

Index Terms—Spatial databases, query processing, authentication, spatial keyword search

1 INTRODUCTION

A spatial keyword query [6], [8], [12], [37], [38] integrates location and text search. Taking a location and a set of keywords as arguments, such queries return relevant spatial web objects that match the arguments. Spatial web objects can be points of interest (e.g., restaurants, tourist attractions, hotels, entertainment services) with a web presence, and they thus have locations as well as textual descriptions. A moving top-k spatial keyword (MkSK) query [39], [40], which takes into account a continuously moving query location, enables a mobile client to be continuously aware of the top-k spatial web objects that best match a query with respect to location and text relevance, and it has numerous mobile uses. For example, a mobile client may activate a “cafe” query in order to be alerted about nearby opportunities for a cup of coffee. With the MkSK query, a client always has an up-to-date result as the client moves. The client can ignore a result and just keep moving until an appealing result appears.

A straightforward solution to the MkSK query is to periodically invoke an existing snapshot spatial keyword query processing technique. However, this approach has the problem that even if snapshot queries are processed very frequently, which is expensive and wasteful because consecutive results are likely to be very similar, there is no guarantee that the user always has the right, up-to-date result. Another possible solution is to extend a buffering technique for spatial kNN query processing [35] to top-k spatial keyword querying. Given a kNN query, this technique retrieves \( k + \Delta k \) nearest neighbors and uses them to derive a buffer region with the property that as long as the user moves inside the region, the kNN result can be derived from the \( k + \Delta k \) objects. However, it is not known how to extend this technique to MkSK queries, where both text relevance and spatial distance are considered. A safe zone based approach [39], [40] for the processing of MkSK queries returns a safe zone to a client together with the query result. The safe zone is a region containing the user’s location and in which the top-k result remains unchanged. The safe zone based approach significantly reduces the communication between clients and the service provider as well as computation costs, since the client needs to request new result only when leaving the safe zone.

Example 1.1: Consider the example MkSK query \( q \) in Figure 1, where the query keywords are “vanilla coffee.” The text relevance of objects \( p_1 \) and \( p_2 \) to \( q \) are 2 and 1, respectively, when using the number of matched keywords for defining relevance. The service provider returns object \( p_2 \) as the top-1 result and the gray circle as the safe zone of \( p_2 \), meaning that as long as the client remains inside the gray circle, object \( p_2 \) is the top-1 result. The curved path shows the client’s movement. When the client crosses the boundary of the gray circle (at \( q’ \)), it sends its updated location to the service provider that then computes and returns a new top-1 result \( p_1 \) and the white region as the safe zone.

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The web is increasingly being used by mobile users, some 20–50% of web queries have local intent [14], [34], and web content with location continues to proliferate (e.g., web pages, business director entries, tweets, check-ins). This development renders it relevant to enable the outsourcing of spatial keyword querying. Such outsourcing of query functionality by an entity that wishes to deliver a service, called a data owner (DO), to a service provider (SP) offers a number of advantages [15], including elastic scaling, cost savings, and high availability when compared to conventional client-server architectures. Moreover, data owners need not buy high performance hardware and hire technical staff to maintain the service. The outsourcing is likely to also improve service availability. In addition, outsourcing can reduce the network latency associated with the delivery of query results.

However, being outside of the administrative scope of the DO, the SP may return inaccurate or incorrect query results, intentionally or unintentionally, e.g., due to cost considerations or invasion of hackers [36]. In our setting, the SP may return an overly large safe zone together with a query result in order to save computing resources and communication bandwidth [27], [43]. The SP may also return an overly small safe zone to incur frequent requests from the client if the SP is paid per request. Therefore, query authentication, i.e., establishing the soundness and completeness of query results at the client side, is highly desirable. Soundness guarantees that every result object is present in the DO’s database and is not tampered with. Completeness ensures that no valid result object is missed in a query result.

Authentication techniques have been developed for a variety of queries, including relational queries [19], [26], [41], sliding window queries [20], [29], spatial queries [16], [42], text similarity queries [27], shortest path queries [43], moving kNN queries [44], moving range queries [45], and subgraph search [11], [30]. However, all existing Authenticated Data Structures (ADS) are either inapplicable or inefficient in relation to our setting, since the authentication of MkSK queries involves verifying both spatial proximity and text relevance. Moreover, authenticating an MkSK query includes verifying both the top-k result and the accompanying safe zone. The safe zone is calculated based on both the objects in the top-k result and the objects not in the top-k result, so that missing a non-result object may cause a safe zone to fail in the authentication. Although authentication techniques for moving kNN queries [44] and moving range queries [45] involve safe zone verification, the safe zone of an MkSK query is very different, i.e., it is a Multiplicatively Weighted (MW) Voronoi cell [25].

To the best of our knowledge, this paper is the first to study the authentication of moving top-k spatial keyword queries. Our proposals also apply to the authentication of static top-k spatial keyword queries (only verifying the top-k result), which is a sub-task of the authentication of MkSK queries. Our contributions include:

- We design a new data structure for the authentication of MkSK queries, the MIR-tree, that enables low computation and communication costs. The MIR-tree is applicable to many variant of the spatial keyword query.
- We design a Verification Object (VO) for authenticating the top-k results and safe zones of MkSK queries. Algorithms for constructing VO and verifying the top-k results and safe zones using VO are developed.
- An enhanced data structure, the MIR*-tree, is proposed to further reduce the communication cost. The idea of the MIR*-tree is applicable for any tree-structured ADS, e.g., the MR-tree [42], where each node contains multiple entries.
- We conduct a thorough experimental study on real data to evaluate the performance of our proposals. The proposed approaches are able to outperform two baseline algorithms exploiting existing authentication techniques by orders of magnitude.

The rest of the paper is organized as follows. The problem setting is presented in Section 2. The two new ADSs, the MIR-tree and the MIR*-tree, the design of a VO, and accompanying algorithms for the authentication of MkSK queries are introduced in Section 3. We describe the two baseline algorithms and study the performance of our proposals in Section 4. Related work is covered in Section 5, and we conclude in Section 6.

2 PROBLEM SETTING

2.1 Moving Top-k Spatial Keyword Query

We consider a generic problem setting in which the future locations of an MkSK query cannot be predicted in advance [39], [40]. Let $D$ be a data set in which each object $p \in D$ is a pair $\langle \lambda, \psi \rangle$ of a point location $p, \lambda$ and a text description, or document, $p, \psi$ (e.g., the facilities and menu of a restaurant). A moving top-k spatial keyword (MkSK) query $q = \langle \lambda, \psi, k \rangle$ takes three arguments: a continuously changing point location $\lambda$, a set of keywords $\psi$, and a number of requested objects $k$. Intuitively, an object whose description is more relevant to the query keywords and is closer to the query location is preferable. We use a weighted distance (Equation 1) [3] as our ranking function. This type of function has been used extensively in the geosciences [13], [18] and it matches the semantics of the query. A key strength of this function is that the different units of measurement of the spatial distance and textual relevance is canceled out in the ratio and thus do not affect the result.

$$rank_q(p) = \frac{||q||}{tr_q(p)},$$

(1)
where \( \|q, p\| \) denotes their Euclidean distance and \( tr_q(p) \) denote the text relevancy of \( p \) to \( q \). The MkSK query retrieves \( k \) objects \( RS \) that minimize the ranking value and a safe zone \( \Upsilon^k(RS) \) within which the top-\( k \) result is valid. Formally, \( \forall p \in RS \ (\forall p' \in D - RS \ (rank_q(p) \leq rank_q(p'))) \) and \( q, \lambda \in \Upsilon^k(RS) \implies RS \) is the top-\( k \) result.

The safe zone of the top-\( k \) result of an MkSK query is an order-\( k \) MW-Voronoi cell [25] that is an irregular geometric entity. The text relevancy of an object to the query is the weight used in the computation of MW-Voronoi cells. We thus use weight \( w(\cdot) \) and text relevancy \( tr_q(\cdot) \) interchangeably. Note that the safe zone preserves the set of top-\( k \) objects, not the ranking of the top-\( k \) objects. More details about safe zones is provided in Appendix A. Table 6 in Appendix A summarises the symbols used in the paper.

2.2 Authentication Framework

Figure 2 illustrates the framework for answering MkSK queries that supports correctness verification. The framework consists of two phases, i.e., initialization and query processing & authentication. In the initialization phase, the DO first gets a private key from a key distribution center. Next, it signs the ADS constructed on the data set using the private key and transfers the ADS and signatures to the SP. A client downloads a public key from the key distribution center and the signatures from the SP. In the query processing and authentication phase, the client first issues an MkSK query. Upon receiving the query, the SP computes the top-\( k \) result, the safe zone, and a verification object \( (VO) \) that encodes the query result and its safe zone. The client gets the \( VO \) from the SP. The top-\( k \) result \( RS \) and its safe zone \( \Upsilon^k(RS) \) are obtained from the \( VO \). The correctness of the top-\( k \) result and the safe zone can be verified by the client using the \( VO \), the signatures, and the public key. The client needs to send a new request to the SP only when it leaves the safe zone.

2.3 Authentication Tasks

The authentication of an MkSK query \( q \) amounts to guaranteeing that the client always has the correct top-\( k \) result while its spatial location changes continuously. The safe zone based approach guarantees that as long as the query does not exit the safe zone, the received top-\( k \) result remains valid. In other words, when the query moves across the boundary of a safe zone, it requests an updated top-\( k \) result and corresponding safe zone. Therefore, authenticating an MkSK query is equivalent to verifying the correctness of both the top-\( k \) result \( RS \) and the corresponding safe zone. The sufficient condition of MkSK query authentication can be further divided into four sub-conditions.

(I) Soundness of a top-\( k \) result.

\[
\forall p \in RS \ (p \in D) \tag{2}
\]

Every object in the top-\( k \) result must present in DO’s data set. Specifically, both the location and the text description of the object are not tampered with and the ranking score \( rank_q(p) \) of \( p \) is computed correctly.

(II) Completeness of a top-\( k \) result.

\[
\forall p \in RS \ (\forall p' \in D - RS \ (rank_q(p) \leq rank_q(p'))) \tag{3}
\]

All objects not in the top-\( k \) result have ranking scores that are no better than any of those of the result objects.

(III) Soundness of a safe zone.

\[
\forall p \in I \ (p \in D) \tag{4}
\]

Since a safe zone is computed from the result objects and the influence objects [40], the facts that the locations and the text descriptions of the influence objects are not tampered with, the text relevancies of influence objects are computed correctly, and sub-condition (I) guarantee the soundness of a safe zone.

(IV) Completeness of safe zone.

\[
\forall \lambda' \in \Upsilon^k(RS) \ (Q(\lambda', \psi, k) = Q(\lambda, \psi, k)) \bigwedge \\
\forall \lambda' \notin \Upsilon^k(RS) \ (Q(\lambda', \psi, k) \neq Q(\lambda, \psi, k)) \tag{5}
\]

For all locations \( \lambda' \) in the safe zone, the result of the query with parameters \( \lambda' \), the keywords of the original query \( \psi \), and \( k \) on the original data set is the same as the result of the original query \( Q(\lambda, \psi, k) \). For all \( \lambda' \) not in the safe zone, the two results are different.

2.4 Threat Model

The SP is the potential adversary [42]. The SP is outside the administrative scope of the DO and thus cannot be trusted. Without the exception of the DO’s private key, adversaries are assumed to know all information, including the public key for the secure-hash function, the ADS, the signatures, and the authentication algorithms. They may alter the data set or the ADS, and they may tamper with the search result.

2.5 Problem Statement

We study the authentication of MkSK queries, aiming for a solution that (i) verifies the soundness and completeness of the top-\( k \) result and the safe zone of a given MkSK query and (ii) optimizes the server-side computational cost and the client/server communication cost, i.e., the \( VO \) size.
3 PROPOSED SOLUTION

Most existing authentication techniques construct a region (range), also called a verification set, that covers the query result. The data objects that fall into the verification set are sent to the client. In addition, summary objects that are used to derive bounds on the ranking scores of the objects outside the verification set are sent to the client. In other words, the VO contains the objects inside the verification set and a number of summary objects. Using this VO, the client is able to authenticate the result. Since the result of an MSK query is retrieved according to Equation 1, authenticating an MSK query involves verifying both the spatial distances and text relevancies of the result objects. The current state-of-the-art solution for authenticating spatial queries, the MR-tree [42], and the existing method for authenticating the query results of text search engines [27] are inefficient because it generates a large VO based on a large verification set, which causes high communication cost and long authentication time, as we will show in the paper’s experimental study. We propose a new ADS, the MIR-tree (Section 3.1), for efficiently verifying MSK query results. It is also applicable to spatial keyword queries based on other ranking functions. The proposed authentication algorithms (in Sections 3.2 and 3.3) construct a compact VO, i.e., put minimum numbers of objects and MIR-tree entries into the VO. Hence, the communication cost and authentication time are saved. To further reduce the size of a VO, we propose an enhanced ADS, the MIR*-tree (in Section 3.4).

3.1 MIR-Tree

The Merkle-IR-tree (MIR-tree) is developed based on the IR-tree [8], by embedding a series of digests in each node of the tree. In the IR-tree, each entry summarizes the spatial distances and text relevancies of the entries in its child node. This index enables the efficient processing of spatial keyword queries, since it is able to prune the search space according to spatial proximity and text relevance simultaneously. In the MIR-tree, to authenticate text relevancies, a word digest is stored with each entry in each posting list in the inverted file attached to each non-leaf node. Formally, for a word $w$, a posting list entry takes the form $(id, weight, h_w(e))$, where $id$ is the identifier of an entry $e$ in a node in the tree, weight is the weight of $w$ in the pseudo text description of $e$, and $h_w(e)$ is the word digest of $e$ for word $w$. The word digest of $e$ for $w$ is computed from the hash value of the concatenation of the binary representation of all the entries in the posting list of $w$ in the child node of $e$:

$$h_w(e) = h((ce_1, weight, h_w(ce_1)) | (ce_2, weight, h_w(ce_2)) | \cdots),$$

(6)

where $ce_i$ ($i = 1, 2, \cdots$) is an entry in the child node of $e$ and $h()$ is a secure hash function. Entries in leaf nodes do not have word digests.

Figure 3 and Table 1 illustrate how word digests are stored and computed, respectively. For example, in InvF-R5 the word digest of word ‘c’ ($h_c(R_1)$) for non-leaf entry $R_1$ is computed as $h_c(R_1) = h((p_1,5) | (p_5,5))$; in InvF-root the word digest of word ‘c’ ($h_c(R_5)$) for non-leaf entry $R_5$ is computed as $h_c(R_5) = h((R_2,7, h_c(R_1)) | (R_3,5, h_c(R_1)))$, which takes the digests in the child nodes ($h_c(R_1)$ and $h_c(R_2)$) into account. The digests of other words are computed similarly. Note that, word digests do not occupy any space in nodes. They are stored in the inverted files attached to nodes. The word digests of the root node (e.g., $h_c(root)$) are signed by the data owner (e.g., RSA($h_c(root)$)) and are stored with the MIR-tree. Each word has a root signature.

Furthermore, to authenticate spatial distances, a spatial digest is stored with each non-leaf entry. Formally, a non-leaf node contains entries of the form $(R_i, R_i.A, R_i.h)$, where $R_i$ is a reference to its child node, $R_i.A$ is the MBR of all rectangles in entries of the child node, and $R_i.h$ is the spatial digest of the child node. The leaf nodes are identical to those of the IR-tree. The spatial digest of a leaf node is computed from the hash value of the concatenation of the binary representation of all objects in the node. The spatial digest of a non-leaf node summarizes its child nodes’ MBRs as well as their digests. The root spatial digest is signed and stored with the MIR-tree. Figure 3 and Table 2 illustrate how spatial digests are stored and computed, respectively.

The shaded parts in Figure 3 show the additions to the MIR-tree over the IR-tree. The update cost of the MIR-tree is comparable to that of the IR-tree. Specifically, when inserting or deleting an object in a leaf node, the spatial and word digests in its parent node are re-computed only if the MBR and the word weights of the parent node are changed. If splitting happens, the spatial and word digests of affected nodes are re-computed. Changes are propagated to the root node.
The proposed ADS, the MIR-tree, has the desirable property that only a minimum number of objects and MIR-tree entries need to be inserted into the \( \mathcal{V}O \). We construct a verification set \( V_{S_r} \) covering the objects with ranking scores smaller than \( \text{rank}_k \) covering the objects with spatial distances smaller than \( 5 \), the maximum spatial distance in the top-\( k \) result. Recall that the smaller the score, the better the ranking. Algorithm 1 shows the pseudo code for constructing the \( \mathcal{V}O \) for the top-\( k \) result. Each entry \( e \) in the \( \mathcal{V}O \) takes the form \((id, \Lambda, H, \{(w, \text{weight}, h_w(\cdot))\})\), where \( id \) is an identifier, \( \Lambda \) is an MBR, \( H \) is a spatial digest, and \( (w, \text{weight}, h_w(\cdot)) \) is a triple where \( w \) is the identifier of one query keyword, \( \text{weight} \) is the weight of word \( w \), and \( h_w(\cdot) \) is the word digest. If \( id \) refers to an object, its \( H \) and \( h_w(\cdot) \) values are empty.

The \( \mathcal{V}O \) is computed by a depth-first traversal of the MIR-tree using the following conditions: (i) if a non-leaf entry \( e \) has a ranking score \( \text{rank}_k(e) = \|q e\|_{\min}/r_q(e) \) that is higher than \( \text{rank}_k \), the \( \mathcal{V}O \) entry for \( e \) is constructed and added to the \( \mathcal{V}O \), and its subtree will not be visited (lines 7–9); otherwise, Algorithm 1 is called to process the child node of \( e \) (lines 25–26); (ii) for any visited leaf node, all the objects in it are added to the \( \mathcal{V}O \) (line 7). When constructing the \( \mathcal{V}O \), a traversal string \( str_{rs} \) is composed that tracks the search in the MIR-tree. It contains the identifiers of the tree entries and objects added to the \( \mathcal{V}O \), as well as the special tokens ‘[‘ and ‘]’ used to mark the scope of a node (line 11). The traversal string is needed in order to avoid having duplicate entries in the \( \mathcal{V}O \), since the \( \mathcal{V}O \) for the top-\( k \) result and the \( \mathcal{V}O \) for the safe zone may otherwise have common entries, as will be explained later. Finally, the \( \mathcal{V}O \) and the traversal string \( str_{rs} \) are sent to the client.

### 3.2.1 \( \mathcal{V}O \) Construction

The proposed ADS, the MIR-tree, has the desirable property that only a minimum number of objects and MIR-tree entries need to be inserted into the \( \mathcal{V}O \). We construct a verification set \( V_{S_r} \) covering the objects with ranking scores smaller than \( \text{rank}_k \), i.e., the maximum score in the top-\( k \) result. Recall that the smaller the score, the better the ranking. Algorithm 1 shows the pseudo code for constructing the \( \mathcal{V}O \) for the top-\( k \) result. Each entry \( e \) in the \( \mathcal{V}O \) takes the form \((id, \Lambda, H, \{(w, \text{weight}, h_w(\cdot))\})\), where \( id \) is an identifier, \( \Lambda \) is an MBR, \( H \) is a spatial digest, and \( (w, \text{weight}, h_w(\cdot)) \) is a triple where \( w \) is the identifier of one query keyword, \( \text{weight} \) is the weight of word \( w \), and \( h_w(\cdot) \) is the word digest. If \( id \) refers to an object, its \( H \) and \( h_w(\cdot) \) values are empty.

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### Algorithm 1 \( \mathcal{V}O \)Construction (MIRtreeNode \( N \), double \( \text{rank}_k \), MkSKQuery \( q \))

1. Append [ to \( str_{rs} \);
2. for each entry \( e \) in \( N \) do
3. if \( N \) is a leaf node then
4. Append \( e \) to \( str_{rs} \);
5. Add \( \mathcal{V}O\text{Entry}(e) \) to \( \mathcal{V}O \);
6. else \( \triangleright \) \( N \) is a non-leaf node.
7. if \( \text{rank}_k(e) > \text{rank}_k \) then
8. Append \( e \) to \( str_{rs} \);
9. Add \( \mathcal{V}O\text{Entry}(e) \) to \( \mathcal{V}O \);
10. else
11. \( \triangleright \) \( \mathcal{V}O \)Construction (\( e \), \( \text{rank}_k \), \( q \));
12. Append ] to \( str_{rs} \);
13. Return \( \mathcal{V}O \) and \( str_{rs} \);

#### Example 3.1: Figure 3 shows the MIR-tree built on the spatial web objects in Figure 4. Let \( q \) be an MkSK query with query keyword ‘c’, where \( k = 1 \). Its top-1 result is object \( p_1 \). The traversal string \( str_{rs} \) is \([[[p_1][p_2][p_3][p_4][p_5]]] \). The \( \mathcal{V}O \) is \{\((\{p_1, p_2, \Lambda, \{(c, 5)\}, (p_2, p_3, \Lambda, \{(c, 5)\}), (p_3, p_4, \Lambda, \{(c, 7)\}), (p_4, p_5, \Lambda, \{(c, 3)\})\), (p_5, \Lambda, \{(c, 3)\})\), (R_3, R_4, R_5, H, \{(c), (R_4, R_5, R_6, H, \{(c, 4, h_c(R_4))\})\})\}. The entries shown using bold font in Figure 3 are added to the \( \mathcal{V}O \). Since object \( p_4 \) and entry
3.2.2 Verifying a Top-k Result

The client extracts the top-k result from the VO and authenticates it. It first computes the ranking scores of the objects in the VO using Equation 1, ranks them, and obtains the top-k objects. Next, the client re-computes the root spatial and word digests using the str and VO (Algorithm 2) and verifies them against the decrypted root spatial and word signatures. The main idea is to re-construct the MIR-tree traversal performed by the server, i.e., guaranteeing the entries in the VO are from the original MIR-tree. Then, it verifies the correctness of the top-k result by checking whether the ranking score computed from Equation 1 of the kth object is no worse than those of all the other entries in the VO.

Example 3.2: Consider the traversal string str and the VO in Example 3.1. The root spatial digest is re-constructed as h(MBR(p1,λ,p2,λ,p3,λ,p4,λ, p8,λ)) h(MBR(p1,λ,p2,λ)p1[p1,λ][p2,λ]) h(MBR(p3,λ,p4,λ, p8,λ)p3[p3,λ][p4,λ][p8,λ][p8,λ]) h(MBR(R3,λ, R4,λ)) h(R3,λ,R3,λ,H). The root word digest of ‘c’ is h(4, (5, h(4, (5, (3, p5, 3))))). The steps needed to compute the spatial and word digests are shown in Figure 5.

3.2.3 Soundness and Completeness

In this section, we prove that our proposal can authenticate the top-k result.

Proof of Soundness.

Suppose that an object p in the result is fake or modified. The re-constructed root spatial and word digests (hash values) cannot be verified against the signatures provided by the data owner, which is detected, since the hash function is one way collision-resistant and p and its word weights must be used by Algorithm 2.

Proof of Completeness.

Let p be one of the top-k objects and let N be the leaf node containing p. If all the entries in N are in the VO, for sure p is in the VO and is included in the top-k result. If N is pruned by the server, the MBR, keyword weights and digests of N are in the VO. The client obtains the top-k result from the VO, which does not include p. Since the re-constructed root spatial and word digests (hash values) equal the decrypted signatures, the ranking score of N is correct and must be better than that of the kth object, which informs the client about a potential completeness violation.

Algorithm 2 (MBR, SpatialDigest, WordWeightList, WordDigestList) Authentication (VerificationObject VO, String str, MskQuery q)

1: spatialstr ← null;
2: wordstrlist ← null;
3: MBR ← null;
4: WordWeightList ← null;
5: WordDigestList ← null;
6: for each word w in the query keywords do
7: w.str ← null;
8: wordstrlist.add(w.str);
9: w.weight ← −1;
10: WordWeightList.add(w.weight);
11: w.digest ← 0;
12: WordDigestList.add(w.digest);
13: while str is not empty do
14: eV ← str.nextEntry();
15: if eV refers to an object then
16: p ← VO.getEntry(eV);
17: MBR ← MBR ∪ p.λ;
18: spatialstr ← spatialstr[p.id][p.λ];
19: for each word w in {w.weight, h(w(·))} do
20: Find the corresponding w in wordstrlist and
21: WordWeightList;
22: w.str ← w.str([p.id, w.weight]);
23: w.weight ← max{w.weight, w.weight};
24: if eV refers to a non-leaf entry in the MIR-tree then
25: e ← VO.getEntry(eV);
26: MBR ← MBR ∪ e.λ;
27: spatialstr ← spatialstr[e.λ][e.λ];
28: for each word w in {w.weight, h(w(·))} do
29: Find the corresponding w in wordstrlist and
30: WordWeightList;
31: w.str ← w.str([w.weight, w.weight]);
32: w.weight ← max{w.weight, w.weight};
33: if eV is p then
34: (mb, sd, wul, wdl) ← Authentication (VO, str, q);
35: MBR ← MBR ∪ mb;
36: spatialstr ← spatialstr[mbr][sd];
37: for each word w in wul and wdl do
38: Find the corresponding w in wordstrlist and
39: WordWeightList;
40: w.str ← w.str([w.weight, h(w(·))];
41: w.weight ← max{w.weight, w.weight};
42: if eV is p then
43: for each word w in WordDigestList and wordstrlist do
44: w.digest ← h(w.str);
45: Return (MBR, h(spatialstr), WordWeightList, WordDigestList);

3.3 Safe Zone Authentication

Recall that a safe zone is defined by influence objects. The challenge of verifying the safe zone is to ensure that the client notices that influence objects are missing if the SP omits some influence objects. Considering the safe zone in Figure 6(a), suppose that the SP only sends influence objects p2 and p4 to the client, omitting p3. Then the client will construct an incorrect (larger) safe zone, namely the gray region in Figure 6(b). The influence objects p2 and p4 are originally from the data set that can pass the verification.
Computing spatial digests

Computing word digests

by checking root signatures. However, the client cannot
determine whether the influence object set provided by
the SP is complete.

We first consider the safe zone of a top-1 object \( p^* \). Let
\( I^+ \) be the set of influence objects of the safe zone of \( p^* \),
where the weight of each influence object exceeds that of
\( p^* \). A threshold \( \tau \) is defined as the minimum value of the
maximum border distances of the influence objects in \( I^+ \),
i.e.,

\[
\tau = \min_{p \in I^+} \text{bord}_{\max}(p^*, p).
\]

Verification set \( VS_{sz} \) of the safe zone of \( p^* \) covers the
objects whose minimum border distances are smaller than
\( \tau \).

As an example, the gray region in Figure 7 is the
safe zone of top-1 object \( p^* \). The influence object set \( I^+ \) contains \( p_1 \) and \( p_4 \). Threshold \( \tau = \text{bord}_{\max}(p^*, C_{p^*,p_1}) \) is
shown as the dashed circle. The verification set \( VS_{sz} \) of
the safe zone covers objects \( p_1 \) and \( p_2 \) that are influence objects.

We proceed to define the verification set of the safe zone
of top-\( k \) result \( RS \). The order-\( k \) MW-Voronoi cell of set
\( RS \), denoted by \( \Upsilon^k(RS) \), contains all locations that take
set \( RS \) as the top-\( k \) result. In other words, \( \Upsilon^k(RS) \) is
the safe zone for the result set \( RS \). Region \( \Upsilon^k(RS) \) can be
represented by the intersections of order-\( k \) MW-Voronoi
cells [25]:

\[
\Upsilon^k(RS) = \bigcap_{p_i^j \in RS} \Upsilon_{D-RS}(p_i^j),
\]

where \( \Upsilon_{D-RS}(p_i^j) \) denotes the MW-Voronoi cell of \( p_i^j \) with
respect to the object set \( D-RS \). For each result object \( p_i^j \),
threshold \( \tau_j \) is computed as \( \tau_j = \min_{p \in I^+_j} \text{bord}_{\max}(p^*_j, p) \),
where \( I^+_j \) is the influence object set of \( p_i^j \). We next give the
formal definition of the verification set of the safe zone.

**Definition 3.3:** The verification set \( VS_{sz} \) of a safe zone of
top-\( k \) result \( RS \) covers the objects that satisfy the following condition:

\[
\bigwedge_{p_i^j \in RS} \text{bord}_{\min}(p_i^j, p) \leq \tau_j
\]

**Theorem 3.1:** The objects covered by the verification set
\( VS_{sz} \) of the safe zone of top-\( k \) result \( RS \) are sufficient to
reconstruct the safe zone.

**Proof:** The theorem implies that the objects outside the
verification region do not contribute the safe zone.

According to Definition 3.3, any object \( p \) outside the
verification set \( VS_{sz} \) satisfies the following condition:

\[
\bigvee_{p_i^j \in RS} \text{bord}_{\min}(p_i^j, p) > \tau_j
\]
Suppose the minimum border distance of \( p \) exceeds the threshold \( \tau \) of one result object \( p^* \), i.e., \( \text{bord}_{\text{min}}(p^*, p) > \tau \). Based on the definition of the minimum and maximum border distances (Definitions 3.1 and 3.2) and \( \tau \) (Equation 9), the dominant region \( \text{Dom}_{p^*, p} \) completely contains the safe zone of \( p^* \), i.e., \( \text{Dom}_{p^*, p} \supset \text{\text{Y}(p^*)} \). In other words, object \( p \) does not contribute to the safe zone of \( p^* \). According to the definition of the safe zone of top-\( k \) result \( \mathcal{RS} \) (Equation 19), object \( p \) does not contribute to the safe zone of the top-\( k \) result.

Having the verification set, the \( \mathcal{VO} \) for the safe zone can be constructed by applying Algorithm 1 with the following modification:

1. The input parameters are: MIRtreeNode \( N \), top-\( k \) result \( \mathcal{RS} \), thresholds \( \{\tau_j\} \).
2. Replace \( s\text{tr}_{rs} \) with the traversal string \( s\text{tr}_{sz} \) for the safe zone.
3. Replace the condition in line 7 with Equation 12, where the minimum border distance of an entry \( e \) in the MIR-tree is computed as \( \text{bord}_{\text{min}}(p^*, e) = \|p^* - e\|_{\text{min}}(w(p^*)/(w(p^*) + w(e))). \)

The \( \mathcal{VO} \) for the top-\( k \) result and the safe zone can be constructed using a single traversal of the MIR-tree. The traversal strings \( s\text{tr}_{rs} \) and \( s\text{tr}_{sz} \) indicate the relevant objects and entries for the top-\( k \) result and the safe zone, respectively, while only one copy of each object and entry are stored in the \( \mathcal{VO} \).

### 3.3.2 Verifying a Safe Zone

The client re-constructs the safe zone using the \( \mathcal{VO} \) and authenticates it. After obtaining the top-\( k \) objects, it calculates the safe zone using the top-\( k \) objects and the remaining objects in the \( \mathcal{VO} \). Next, the client re-computes the root spatial and word digests using the \( s\text{tr}_{sz} \) and the \( \mathcal{VO} \) by applying Algorithm 2, and it verifies them against the decrypted root spatial and word signatures, making sure that the entries in the \( \mathcal{VO} \) are from the original MIR-tree. Then, it verifies the correctness of the safe zone by checking whether all the other entries in the \( \mathcal{VO} \) except the top-\( k \) objects and influence objects satisfy Equation 12.

### 3.3.3 Soundness and Completeness

In this section, we prove that our proposal can authenticate a safe zone.

**Proof of Soundness.**

Suppose that the safe zone constructed by the client is fake or modified, i.e., the influence objects derived from the \( \mathcal{VO} \) are fake or modified. The re-constructed root hash values cannot be verified against the root signatures provided by the data owner, which is detected, since the hash function is one way collision-resistant and the influence objects and their word weights must be used by Algorithm 2.

**Proof of Completeness.**

Let \( p \) be one of the influence objects and let \( N \) be the leaf node containing \( p \). If all the entries in \( N \) are in the \( \mathcal{VO} \), for sure \( p \) is in the \( \mathcal{VO} \) and will be used to construct the safe zone. If \( N \) is pruned by the server, the MBR, keyword weights and digests of \( N \) are in the \( \mathcal{VO} \). The client obtains the safe zone from the \( \mathcal{VO} \), without using \( p \). Since the re-constructed root hash values match the root signatures, the dominant regions of the top-\( k \) result object over \( N \) is correct and must intersect the safe zone, which informs the client about a potential violations of completeness.

### 3.4 MIR*-Tree

The \( \mathcal{VO} \) size is critical to minimize the client/server communication cost and reduce the client-side result verification time. We proceed to propose an enhanced ADS, the MIR*-tree, that enables a reduction of the \( \mathcal{VO} \) size. Specifically, the digests in the MIR*-tree are computed differently than in the MIR-tree. The underlying idea is applicable to any tree structure based ADS, e.g., the MR-tree [42], where each node contains multiple entries.

In the MIR-tree, each non-leaf entry has one spatial digest and several word digests. When a non-leaf entry (that is pruned) is added to the \( \mathcal{VO} \), its MBR, one spatial digest, and \( m \) word digests are included, where \( m \) is the number of query keywords. Hence, for a visited non-leaf node in the MIR-tree, \( n(m + 1) \) digests may be added to the \( \mathcal{VO} \), where \( n \) is the number of pruned entries. Let \( S_d \) be the size in bytes of a digest and let \( S_m \) be the size in bytes of an MBR. The contribution of a non-leaf node \( N \) to the \( \mathcal{VO} \) is \( S_N = n[(m + 1)S_d + S_m] \). With a node fanout in the range of hundreds, \( n \) can be large. And the size of a digest \( S_d \) is large (e.g., 64 bytes when using the SHA-512 hash function), while the size of an MBR \( S_m \) is at most 32 bytes if using double precision. Hence, the \( \mathcal{VO} \) size is dominated by the number of digests included.

We propose a new way to compute the digests by incorporating the Rabin cryptosystem (quadratic residue) [31] to reduce the digests being added to the \( \mathcal{VO} \), resulting in a new index structure, the MIR*-tree, where for each visited non-leaf node, at most \( m + 1 \) digests are added to the \( \mathcal{VO} \). Then the contribution of a non-leaf node \( N \) in the MIR*-tree to the \( \mathcal{VO} \) is \( S_N' = (m + 1)S_d + n \cdot S_m \). Given an \( n \) value larger than 100, the \( \mathcal{VO} \) size of the MIR*-tree can be significantly reduced, compared with that of the MIR-tree.

Finding a solution to \( x^2 \equiv y \pmod{N} \), where \( N = pq \) and \( p, q \) are primes, is equivalent to factoring a large number [31]. Thus, finding \( x \) under passive attacks is as hard as factoring. We proceed to exemplify how to compute the digests in the MIR*-tree using the quadratic residue. If an entry points to a leaf node, its digests are the same as those in the MIR-tree. If an entry points to a non-leaf node, its spatial digest is computed from the hash value of the concatenation of the binary representation of all the MBRs in its child node and one digest that is the modulo multiplication of all the digests’ squares in its child node. As shown in Figure 8, the child node of \( R_5 \) contains four entries. The spatial digest of \( R_5 \) is computed as \( R_5.H = h[R_5.A|R_5.B|R_5.C|R_5.D | H_5] \), where \( H_5 = \prod_{i=1}^{4} R_i.H^2 \pmod{N} \). The word digests are computed in a similar way. Suppose all the five entries in Figure 8 contain word ’c’ with weight 1. The word digest
of $R_5$ for word ‘c’ is $h_c(R_5) = h((R_1, 1)(R_2, 1)(R_3, 1)(R_4, 1))|H_w)$, where $H_w = \prod_{i=1}^{4} h_c(R_i)^2 \mod N$.

![Spatial Digits in the MIR*-Tree](image)

Fig. 8. Spatial Digits in the MIR*-Tree

During the process of constructing a $\mathcal{VO}$, when visiting a non-leaflet node, the modulo multiplication of the spatial digits of pruned entries and the modulo multiplication of the word digits of pruned entries for each query keyword are added to the $\mathcal{VO}$. Consider the example in Figure 8, where all the five entries contain word ‘c’. If entries $R_1, R_2, \text{and } R_4$ are pruned, we add one spatial digit $H_1' = \prod (R_1, H, R_2, H, R_4, H) \mod N$ and one word digit $H_2' = \prod (h_c(R_1), h_c(R_2), h_c(R_4)) \mod N$ to the $\mathcal{VO}$. In contrast, in the MIR-tree, three spatial digits, $R_1, H, R_2, H, \text{and } R_4, H$, and three word digits, $h_c(R_1), h_c(R_2), \text{and } h_c(R_4)$, are added to the $\mathcal{VO}$.

The client is able to compute the spatial digit and the word digit for a node in a similar way as does the server using the $\mathcal{VO}$. Continuing with the above example, the client first re-computes the spatial digit and the word digit of $R_4$, i.e., $R_1, H$ and $h_c(R_4)$. Then $H_2 = (H_1' - R_4, H)^2 \mod N$ and $H_w = (H_2' - h_c(R_4))^2 \mod N$. The correctness is guaranteed by $\prod h_i^2 \mod N = (\prod h_i)^2 \mod N$.

To observe the potential improvement, Figure 9(a) plots $S_N$ and $S_N^*$ as a function of $n$, where $S_m = 32$, $S_d = 64$, and $m = 1$. As $n$ increases, i.e., more entries in a non-leaflet node are pruned, the difference between $S_N$ and $S_N^*$ becomes larger. The size of the contribution of a non-leaflet node in the MIR*-tree to the $\mathcal{VO}$ ($S_N^*$) increases more slowly than that in the MIR-tree ($S_N$). Figure 9(b) illustrates that $S_N$ and $S_N^*$ exhibit similar trends as the number of query keywords $m$ increases. We expect that the MIR*-tree produces a smaller $\mathcal{VO}$ than does the MIR-tree, and the empirical study confirms our expectation.

![Fig. 9. $S_N$ vs. $S_N^*$, $S_m = 32$, $S_d = 64$](image)

4 EXPERIMENTAL STUDY

We conduct empirical studies to evaluate our proposals, comparing with two baselines introduced in Section 4.1. Section 4.2 presents the data sets, queries, parameters, and platform used in experiments. The proposals for the authentication of M$k$SK queries are evaluated in Section 4.3.

4.1 Baselines

We develop two baseline algorithms that exploit existing techniques for the authentication of M$k$SK queries. One is Inverted Index and Sorted Lists (IISL) utilizing signature chaining [26], and the other is Inverted Index and MR-tree (IMR) that combines existing proposals [27, 42].

**Inverted Index and Sorted Lists (IISL).**

The IISL consists of two parts. One part is the inverted index [46], which is an efficient and popular index for keyword search, indexing the text descriptions of objects. The other part contains two sorted lists, indexing the spatial point locations of the objects. One list is for $x$-coordinates and the other one is for $y$-coordinates. Both lists are sorted ascendingly. We apply the idea of signature chaining to authenticate M$k$SK queries. A signature chain is constructed for each of the two sorted lists and each posting list in the inverted index.

Given an M$k$SK query $q$, we first compute its top-$k$ result $R_S$ and the safe zone $\mathcal{Y}^k(R_S)$. In order to utilize the signature chaining to authenticate $q$, verification sets $VS_X$ and $VS_Y$ for the two sorted lists of coordinates and $VS_w$, for the posting list of each query keyword in the inverted index are constructed to compute a $\mathcal{VO}$. We show how to compose the above verification sets in the appendix.

A $\mathcal{VO}$ is constructed by retrieving the objects covered by the verification sets. Each entry in the $\mathcal{VO}$ takes the form $(id, \Lambda, \{w, weight\})$, where $id$ is the identifier, $\Lambda$ is the minimum bounding rectangle, and $(w, weight)$ is a pair of a query keyword and its weight. In addition, a condensation of an RSA signature is computed by multiplying all the signatures of the objects in the $\mathcal{VO}$. The client recomputes the top-$k$ result and the safe zone from the $\mathcal{VO}$ and verifies the condensed RSA signature.

**Inverted Index and MR-tree (IMR).**

The IMR consists of an inverted index that indexes the text descriptions of the objects and an MR-tree that indexes the spatial point locations of the objects. Verification sets $VS_T$ and $VS_s$, presented in the appendix are used to construct a $\mathcal{VO}$ to authenticate an M$k$SK query result. Specifically, the efficient approach TNRA-Chain-MHT proposed by Pang et al. [27] is applied to the inverted index to authenticate the text relevancies of objects using $VS_T$, and the MR-tree authenticates the spatial distances of objects using $VS_S$.

4.2 Experiment Setup

We use two data sets, each containing objects with a point location and a text description, for studying the robustness and performance of the proposals. Data set EURO contains points of interest (e.g., ATMs, hotels, stores) in Europe¹. Data set LONDFLI contains objects referring to photos taken in the area of London, where each object is composed by a spatial location and a text describing the photo (title and description). Table 3 offers additional details. We normalize object locations to fit a square domain with side length 10,000 meters.

The Brinkhoff generator [5] is used to generate a trajectory using one location acquisition per timestamp (second). Each experiment has 100 moving queries with such trajectories. The keyword set of each query is generated by randomly picking an object in the data set and randomly choosing adjacent words from the document of the object.

We generally report the average value per query per timestamp of the following: (i) server-side elapsed time (for the query processing and the VO construction), (ii) server-side simulated I/O cost, (iii) client-side elapsed time (for the result verification), and (iv) communication cost (the VO size in KBytes).

We study the two baselines and our proposed solutions—the MIR-tree and the MIR*-tree. By default, we set the number of results to 1, the number of query keywords to 2, the client moving speed to 10 m/s, and the fanout of the MIR-tree and the MIR*-tree to 200. We used disk-based index structures with a page size fixed at 4 KBytes. A simulated LRU buffer with size 256MB is used. All the algorithms are implemented in Java and run on a Linux machine with one processor (Intel(R) Core(TM)2 Quad CPU Q8400 @ 2.66 GHz) and 4 GB memory. The Java Virtual Machine Heap is set to 2 GB. We employ the SHA-512 as the hash function and 1024-bit RSA as the signature scheme.

### 4.3 Performance Results

We study the proposed methods under varying settings, including the performance on different data sets, varying number of requested results $k$, varying numbers of query keywords, varying speeds of moving queries, varying fanouts of the MIR-tree and the MIR*-tree, and varying data set sizes.

#### Evaluation on Different Data Sets.

Table 4 reports the average server elapsed time, server I/O cost, client elapsed time, number of entries in the VO, VO size, index size, and communication frequency for the four methods on the two data sets. As expected, the server elapsed time is proportional to the server I/O cost and the client elapsed time is correlated with the VO size. The MIR variants, including the MIR-tree and the MIR*-tree, outperform baseline IISL by orders of magnitude in all aspects. The reason why IISL has extremely poor performance is that its text and spatial score intervals are derived separately. Furthermore, its spatial score interval is converted into coordinate intervals on the $x$ and $y$ axes. Those intervals cover too many objects that are added to the VO. Hence, its processing costs at both the server and client are high, and the VO size is large.

The server elapsed time and I/O cost of baseline IIMR are comparable to those the MIR variants, while the MIR variants outperform the IIMR by orders of magnitude in terms of the client elapsed time, number of entries in the \( \mathcal{V}_O \), and the VO size. Although the IIMR also derives the text score interval and the spatial score interval separately, its processing cost is reduced due to the use of the MR-tree. However, the separate intervals still cover too many objects that are added to the VO, resulting in high verification cost at the client and large VO size.

The performance of the MIR-tree and the MIR*-tree are comparable in terms of server elapsed time and I/O cost. The numbers of entries in the VO for the MIR-tree and the MIR*-tree are the same. This is because the two indexes have similar structures and processing algorithms. The MIR*-tree differs from the MIR-tree in how digests are computed. In the MIR*-tree, fewer digests are added to the VO so that its client elapsed time and the VO size are significantly smaller than that for the MIR-tree.

In terms of index size, the IIMR is the smallest. The MIR-tree and the MIR*-tree occupy more space than does the IIMR due to the inverted files attached to non-leaf nodes. The IISL occupies substantial space due to the signature chains on the sorted lists of the $x$ and $y$ coordinates. These indexes, except for the IIMR, cannot fit in main memory.

Our proposals outperform the two baseline algorithms consistently on the two data sets. We use LONDFLI as the default data set for the remaining experiments. Baseline IISL is omitted due to its poor performance and high storage cost.

### Table 3

<table>
<thead>
<tr>
<th>data set</th>
<th># of objects</th>
<th># of distinct words</th>
<th>average # of words per object</th>
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</thead>
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<tr>
<td>EURO</td>
<td>162,033</td>
<td>35,315</td>
<td>18</td>
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<tr>
<td>LONDFLI</td>
<td>1,255,149</td>
<td>222,613</td>
<td>8</td>
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### Table 4

<table>
<thead>
<tr>
<th>Data Set</th>
<th>EURO</th>
<th>LONDFLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Server elapsed time (milliseconds)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IISL</td>
<td>215.73</td>
<td>48966.67</td>
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<td>IIMR</td>
<td>20.03</td>
<td>169.69</td>
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<td>MIR</td>
<td>20.88</td>
<td>211.53</td>
</tr>
<tr>
<td>MIR*</td>
<td>14.09</td>
<td>191.03</td>
</tr>
<tr>
<td>Server I/O cost (page accesses)</td>
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<td></td>
</tr>
<tr>
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<td>2569.29</td>
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<td>IIMR</td>
<td>1.80</td>
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<tr>
<td>Client elapsed time (milliseconds)</td>
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<tr>
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<td>97.26</td>
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<tr>
<td># of entries in the VO</td>
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<td>Index size (GB)</td>
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<tr>
<td>MIR*</td>
<td>3.13</td>
<td>23.67</td>
</tr>
</tbody>
</table>

### Varying $k$

Figure 10 shows the performance of the three approaches when varying the number $k$ of requested results on data set LONDFLI. The MIR variants substantially and consistently
outperform the baseline in terms of client elapsed time and \( \mathcal{VO} \) size. As expected, the server elapsed time, client elapsed time, and \( \mathcal{VO} \) size of the three methods increase as \( k \) increases, since requesting more result objects increases the computation cost at the server side and calls for a larger \( \mathcal{VO} \), resulting in higher verification cost at the client side. The server I/O costs of the three methods decrease slightly as \( k \) increases. This is mainly because more result objects produce a smaller safe zone, which enables more pruning. Hence, fewer index nodes (pages) are loaded from disk. However, the processing cost of each visited index node increases due to the larger result set.

**Fig. 10. Performance per Timestamp, Varying \( k \)**

**Varying the Number of Query Keywords.**

Figure 11 shows the server elapsed time, server I/O cost, client elapsed time, and \( \mathcal{VO} \) size of the three methods when varying the number of query keywords on data set LOND-FLI. The MIR variants again consistently beat the baseline algorithm. As the number of query keywords increases, the performance of the three approaches decreases, since more objects are involved in the computations.

**Fig. 11. Performance per Timestamp, Varying the Number of Keywords**

**Varying the Speed of Moving Queries.**

Figure 12 shows the performance of the three approaches when varying the speed of the moving queries on data set LOND-FLI. We observe that as the speed increases, the computation and communication costs increase slightly, since a query with high speed more quickly leaves the safe zone and requests a query re-evaluation from the server. The MIR variants outperform the baseline significantly at different speeds.

**Fig. 12. Performance per Timestamp, Varying the Speed of Moving Queries**

and the server I/O cost of the MIR-tree and the MIR*-tree are comparable, while the MIR*-tree has lower client elapsed time and smaller \( \mathcal{VO} \) size than the MIR-tree. The reason is that in the MIR*-tree, fewer digests are added to the \( \mathcal{VO} \), thus reducing the processing cost at the client and the \( \mathcal{VO} \) size. And the operation of deriving one digest from multiple digests does not add much computational cost in the MIR*-tree. As the fanout increases, the server elapsed time and I/O cost decrease, since the heights of the MIR-tree and the MIR*-tree decrease so that it is faster to reach the leaf level to obtain results. In contrast, the client elapsed time and \( \mathcal{VO} \) size increase as the fanout increases. Since a node contains more entries, more entries may be added to the \( \mathcal{VO} \), it results in higher processing cost at the client side. The \( \mathcal{VO} \) size of the MIR*-tree is 20% smaller than that of the MIR-tree when the fanout is 200. The improvement of the MIR*-tree decreases as the fanout increases. That is mainly because the number of pruned entries decreases.
In general, existing query authentication techniques are classified into MHT-based approaches and approaches using signature chaining.

**MHT-based Approaches.** A popular authentication approach based on the MHT [23] has been used widely in many proposals [19], [27], [42], [43]. It is a hierarchical structure in which digests of nodes are recursively computed using a secure hash function from the leaf level to the root level. Then, the DO signs the root digest using a private key, e.g., RSA [33].

Devanbu et al. [9] authenticate one-dimensional range queries using an MHT on the query attribute. To prove the completeness of the result for a range query \([a, b]\) over a sorted list \((r_1, r_2, \ldots, r_n)\), the SP discloses the boundary records to the client. In other words, the query result is expanded into \((r_{i-1}, r_i, \ldots, r_j, r_{j+1})\), where \(r_{i-1} < a \leq r_i, r_j \leq b < r_{j+1}\), and \(1 < i < j < n\). The boundary records guarantee that no records in the query range are omitted. In addition to the expanded result, for each node visited in the MHT, digests of all left-siblings of the left boundary record \(r_{i-1}\) and digests of all right-siblings of the right boundary record \(r_{j+1}\) on the paths from the root are added to the \(\mathcal{VO}\). If the re-constructed root digest from the \(\mathcal{VO}\) matches the decrypted root signature, the result is sound. Later, two disk-based adaptations of the MHT, i.e., the VB-tree [28] and the Merkle B-tree (MB-tree) [19], have been proposed.

Pang et al. [27] apply the MHT to the inverted index and documents to authenticate text retrieval query results (keyword search). The proposed Term-MHT builds an MHT on the posting list of each word, while the Term-CMHT constructs an MHT on the posting list entries in each disk page, since a posting list normally occupies several disk pages. The digest of the MHT (hash value of the root) for a disk page is integrated into the MHT for the previous disk page. The digest of the first page of each posting list is signed by the DO, which is used to verify any \(j\) leading pages of a posting list. A document is represented by a sequence of entries \((w, \text{weight})\) in ascending \(w\) order. The proposed Document-MHT takes the document entries as leaves. Both the Term-MHT and Term-CMHT are used to guarantee that posting lists involved in the query processing are correct, and the Document-MHT is used to check the computed score of each encountered document.

Another solution inspired by the MHT technique is the MR-tree [42], an index based on the \(R^*\)-tree that is capable of authenticating arbitrary spatial queries. Its leaf nodes are identical to those of the \(R^*\)-tree. Non-leaf nodes contain entries of the form \(e = (p, A, H)\), where \(p\) is a reference to a child node, \(A\) is the MBR of all rectangles in entries of the child node, and \(H\) is a digest of the child node. The digest of a leaf node is computed from the hash value of the concatenation of the binary representation of all objects in the node. The digest of a non-leaf node summarizes its child nodes’ MBRS as well as their digests. The digest of the root node is signed by the DO and is stored with the tree.

For the client to verify a query result, the SP utilizes the MR-tree to generate a verification object \(\mathcal{VO}\). A circular verification set \(\oplus(q, \gamma)\) with center \(q\) and radius equal to the distance between \(q\) and its \(k\)th nearest neighbor is defined.

### Table 5

<table>
<thead>
<tr>
<th>Dataset Size</th>
<th>MIR</th>
<th>MIR*</th>
</tr>
</thead>
<tbody>
<tr>
<td>200K</td>
<td>4.03</td>
<td>7.91</td>
</tr>
<tr>
<td>400K</td>
<td>7.91</td>
<td>12.13</td>
</tr>
<tr>
<td>600K</td>
<td>12.13</td>
<td>15.76</td>
</tr>
<tr>
<td>800K</td>
<td>15.76</td>
<td>19.71</td>
</tr>
<tr>
<td>1M</td>
<td>19.71</td>
<td>19.71</td>
</tr>
</tbody>
</table>

### 5 Related Work

#### 5.1 Static Query Authentication

In general, existing query authentication techniques are classified into MHT-based approaches and approaches using signature chaining.
The MR-tree is traversed in depth-first manner with the following conditions: (1) a non-leaf entry $e$ is added to the Voronoi $\mathcal{V}O$ if $e$ does not intersect $\odot(q, \gamma)$; (2) all objects in a visited leaf node are added to the Voronoi $\mathcal{V}O$. To authenticate the query result, the client first reconstructs the digest of the root of the MR-tree from $\mathcal{V}O$. Then it verifies it against the root signature offered by the data owner. Then the client finds the $k$-NNs directly from the Voronoi $\mathcal{V}O$, defines its own verification set $\odot(q, \gamma')$, and checks whether no non-leaf entries in Voronoi $\mathcal{V}O$ intersects $\odot(q, \gamma')$. If yes, the correctness of the NN result is guaranteed.

Lin et al. [22] study the authentication of location-based skyline queries. They propose two authentication methods. One is based on the MR-tree. The other one utilizes the MR-Sky-tree that is designed especially for skyline queries.

**Signature Chaining.** Unlike MHT, signature chaining [26] authenticates both the soundness and completeness of one-dimensional range queries. Consider a data set containing 4 records $p_1$-$p_4$, sorted on the query attribute. The data owner inserts two special records $p_0$ and $p_5$ with values $\rightarrow \infty$ and $-\rightarrow \infty$. A signature is created for each record, except the special records. The signature of a record $p_i$ is based on the hash value of the concatenation of its preceding record $p_{i-1}$, itself $p_i$, and its succeeding record $p_{i+1}$. The data set and the signatures are sent to the SP. Assume that the result of a range query contains $p_2$, $p_3$, and $p_4$. The SP returns a Voronoi containing the result $(p_2, p_3, p_4)$, the signature of each record in the result $(\text{sig}_{123}, \text{sig}_{234}, \text{sig}_{345})$, and the boundary records $p_1$ and $p_5$. In order to reduce the Voronoi size, the Condensed-RSA [24] adds only one signature $\text{sig} = \text{sig}_{123} \cdot \text{sig}_{234} \cdot \text{sig}_{345} \mod n$ to the Voronoi instead of three. Given a Voronoi, the client first checks that the signature is valid, which guarantees that all the records in the Voronoi are correct and adjacent. Then the client checks that the two boundary records fall outside the query range, which ensures that no result record is missing at the range boundaries, i.e., $p_2$ and $p_4$ are indeed the first and last records of the result.

This scheme has also been extended to multi-dimensional index structures [7]. Compared with the MHT-based approaches, signature chaining methods incur high index construction costs, storage overhead, and client-side verification time [19].

The two baseline algorithms considered in the experimental study exploit the above authentication techniques (MHT, MR-tree, and signature chaining), and they are found to be inefficient compared to our proposals.

### 5.2 Moving Query Authentication

Existing technique for authenticating moving $k$-NN queries [44] cannot help in authenticating $M_k$-SK queries, since the safe zone of a $k$-NN query is a Voronoi cell [25], while the safe zone of an $M_k$-SK query is a Multiplicatively Weighted Voronoi (MW-Voronoi) cell [25]. A Voronoi cell considers only spatial distance and has a polygon shape. In contrast, an MW-Voronoi cell defines the influence region of an object based on its weight (e.g., text relevance) and is an irregular shape consisting of edges, arcs and holes. The structures and construction algorithms for these different kinds of cells are totally different. Figure 15(b) illustrates the MW-Voronoi diagram of the same nine objects as in Figure 15(a). In the MW-Voronoi diagram, each object has a weight (underlined numbers). These weights are the text relevencies between the objects and the query, known only when the query is submitted. In other words, different queries produce different MW-Voronoi diagrams on the same data set. Thus, pre-computation based approaches [17], [44] are inapplicable, since enumerating all possible MW-Voronoi diagrams is computationally prohibitive.

![Fig. 15. Voronoi Versus MW-Voronoi Diagrams](image)

Yung et al. [45] study the authentication of moving range queries. The safe zone of a moving range query and its verification set are also different from ours, and thus their technique is inapplicable to our problem.

### 6 Conclusion

This paper proposes a new authenticated data structure, the MIR-tree, to efficiently authenticate moving top-$k$ spatial keyword query results, thus guaranteeing the soundness and completeness of both a top-$k$ result and its corresponding safe zone. A verification object for authenticating the top-$k$ results and safe zones of $M_k$SK queries is designed. Algorithms for constructing and using verification objects for verifying the top-$k$ results and safe zones are developed. An enhancement of the MIR-tree, the MIR$^*$-tree, is proposed to further reduce the communication cost. Extensive empirical studies on real data sets demonstrate that the proposed approaches are capable of outperforming two baseline algorithms that utilize existing techniques by orders of magnitude.

### References


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**APPENDIX A**

**SAFE ZONE**

A safe zone or the query considered is an order-\(k\) multiplicatively weighted Voronoi (MW-Voronoi) region [25]. Here, we recall relevant definitions.

Let \(\mathcal{D}\) be a set of weighted points in two-dimensional Euclidean space \(\mathcal{U}\). A point \(p\) in \(\mathcal{D}\) has (i) a weight \(w(a)\) and (ii) coordinates \((a_x, a_y)\). The weighted distance between any point \(z\) in \(\mathcal{U}\) and \(p\) is defined as \(d_w(z, p)\), where \(\|z - a\|\) is the Euclidean distance between \(z\) and \(a\).

**Definition A.1:** The dominant region of point \(a\) over point \(b\) is defined as:

\[
\text{Dom}_{a,b} = \{z \in \mathcal{U} \mid d_w(z, a) \leq d_w(z, b)\}. \tag{13}
\]

To characterize the shape of \(\text{Dom}_{a,b}\), we define the Apollonius circle and explain its relationship with \(\text{Dom}_{a,b}\). As an example, Figure 16 shows the Apollonius circle of two points \(a\) and \(b\). Any location \(z\) on the circle satisfies the equation \(\|z - a\| = \|z - b\|\).

**Definition A.2:** Given two points \(a\) and \(b\) and a constant \(0 \leq \mu \leq 1\), the Apollonius circle \(C_{a,b}\) is defined as all locations \(z\) that satisfy: \(\|z - a\| = \mu \cdot \|z - b\|\). The Apollonius circular region \(C_{a,b}\) is defined as the region where \(\|z - a\| \leq \mu \cdot \|z - b\|\).

**Lemma A.1:** [Radius and center, Apollonius circle]

Given two points \(a\) and \(b\) such that \(w(a) < w(b)\), \(\text{Dom}_{a,b} = C_{a,b}\). The center \(o_{a,b}\) and the radius \(r_{a,b}\) of \(C_{a,b}\) are shown in Equations 14 and 15. In addition, we have: \(\text{Dom}_{b,a} = \mathcal{U} - C_{b,a}\), where \(\mathcal{U}\) is the spatial domain.

\[
o_{a,b} = \left(\frac{w^2(b)a_x - w^2(a)b_x}{w^2(b) - w^2(a)}, \frac{w^2(b)a_y - w^2(a)b_y}{w^2(b) - w^2(a)}\right) \tag{14}
\]

\[
r_{a,b} = \frac{w(a) - w(b)\|o_{a,b}\|}{w^2(b) - w^2(a)} \tag{15}
\]

We will use “circle” and “circular region” interchangeably.

For the special case \(w(a) = w(b)\), the Apollonius circular region \(C_{a,b}\) degenerates to the perpendicular bisector plane \(\perp_{a,b}\). In general, the dominant region \(\text{Dom}_{a,b}\) is expressed as:

\[
\text{Dom}_{a,b} = \begin{cases} 
C_{a,b} & \text{if } w(a) < w(b) \\
\mathcal{U} - C_{b,a} & \text{if } w(a) > w(b) \\
\perp_{a,b} & \text{if } w(a) = w(b) 
\end{cases} \tag{16}
\]

The MW-Voronoi diagram of \(\mathcal{D}\) is the collection of MW-Voronoi regions of all points in \(\mathcal{D}\). These regions form a (disjoint and complete) partitioning of the spatial domain.

**Definition A.3:** Given a (result) point \(p^* \in \mathcal{D}\), its MW-Voronoi region with respect to \(\mathcal{D}\) is defined as:

\[
\Upsilon(p^*) = \bigcap_{p \in \mathcal{D} - \{p^*\}} \text{Dom}_{p^*,p} \tag{17}
\]

Based on a (result) point \(p^* \in \mathcal{D}\), we can partition \(\mathcal{D}\) into \(\mathcal{D} = \mathcal{D}^+ \cup \mathcal{D}^- \cup \mathcal{D}^0 \cup \{p^*\}\), where set \(\mathcal{D}^+\) contains all points with higher weight than \(p^*\), set \(\mathcal{D}^-\) contains all points with lower weight than \(p^*\), and set \(\mathcal{D}^0\) contains the objects whose weights are identical to \(p^*\). By applying Equation 16, we can re-express the MW-Voronoi region as Equation 18. Thus, higher-weight neighbors add to the MW-Voronoi region of a point, forming convex edges; lower-weight neighbors subtract from it, forming concave edges; and equal-weight neighbors crop it with straight lines.

\[
\Upsilon(p^*) = \bigcap_{p_j \in \mathcal{D}^+} \text{Dom}_{p^*,p_j} \tag{18}
\]

In the example in Figure 17, the gray region is the safe zone (MW-Voronoi region) \(\Upsilon(p_1)\) for the top-1 result \(p_1\). It is calculated by \(C_{p_1,p_4} \cap C_{p_1,p_5} \cap \perp_{p_1,p_2} - C_{p_3,p_1}\). It is generally far from every data object that contributes to defining a safe zone. Note that safe zone \(\Upsilon(p_1)\) consists of 3 arcs (edges) that originate from the Apollonius circles \(C_{p_1,p_4}\) and \(C_{p_1,p_5}\) and the perpendicular bisector \(\perp_{p_1,p_2}\). Apollonius circle \(C_{p_1,p_5}\) does not contribute to the safe zone \(\Upsilon(p_1)\) because \(\Upsilon(p_1)\) is completely inside \(C_{p_1,p_5}\). The objects that are necessary for representing a safe zone are called influence objects. In Figure 17, objects \(p_2\), \(p_3\), and \(p_4\) are influence objects, while \(p_5\) is not.

Let \(\mathcal{R}\) be the top-\(k\) result of a query \(q\). According to Okabe et al. [25], the order-\(k\) MW-Voronoi region of the set \(\mathcal{R}\), denoted by \(\Upsilon^k(\mathcal{R})\), contains all locations that take the set \(\mathcal{R}\) as the top-\(k\) result. In other words, \(\Upsilon^k(\mathcal{R})\) is the safe zone for the result set \(\mathcal{R}\). Region \(\Upsilon^k(\mathcal{R})\) can be expressed by the intersections of order-\(1\) MW-Voronoi regions [25]

\[
\Upsilon^k(\mathcal{R}) = \bigcap_{p^* \in \mathcal{R}} \Upsilon_{\mathcal{D} - \mathcal{R}}(p^*_1), \tag{19}
\]

where \(\Upsilon_{\mathcal{D} - \mathcal{R}}(p^*_1)\) denotes the MW-Voronoi region of \(p^*_1\) with respect to the object set \(\mathcal{D} - \mathcal{R}\).

**APPENDIX B**

**VERIFICATION REGIONS FOR BASELINES**

Given the top-\(k\) result \(\mathcal{R}\) of \(q\), let \(T_k\) be a text score interval \([t_{s_k}, +\infty)\) and \(S_k\) be a spatial score interval \([0, s_{s_k}]\), where \(t_{s_k} = \min_{p \in \mathcal{R}} \{t_q(p)\}\) refers to the minimum text relevance of the objects in \(\mathcal{R}\) and \(s_{s_k}\) refers to the
maximum distance between the query and the result objects, i.e., \( \max_{p \in \mathcal{RS}} (||q_p||) \).

Given the safe zone of the top-k result \( \mathcal{T}^k(\mathcal{RS}) \), let \( I \) be the influence object set of the safe zone. We define two kinds of distances. One is the minimum border distance of an object \( p \) from the result set \( \mathcal{RS} \), which is the minimum value of the minimum border distances of \( p \) from all the result objects in \( \mathcal{RS} \), i.e., \( \text{bord}_{\text{min}}(\mathcal{RS}, p) = \min_{p' \in \mathcal{RS}} \{ \text{bord}_{\text{min}}(p^*_j, p') \} \). The other is the maximum distance between the result set \( \mathcal{RS} \) and an object \( p \), i.e., \( ||\mathcal{RS}||_{\text{max}} \). Consider the non-influence object \( p_s \in (\mathcal{D} \setminus \mathcal{RS} \setminus I) \) with the smallest minimum border distance from \( \mathcal{RS} \), i.e., \( \forall p \neq p_s \land p \in (\mathcal{D} \setminus \mathcal{RS} \setminus I), \text{bord}_{\text{min}}(\mathcal{RS}, p_s) < \text{bord}_{\text{min}}(\mathcal{RS}, p) \). We construct a text score interval \( T_{sz} = (\text{ts}_{sz}, +\infty) \) and a spatial score interval \( S_{sz} = [0, ss_{sz}] \), where \( \text{ts}_{sz} = tr_q(p_s) \) is the text relevance of \( p_s \) and \( ss_{sz} = ||\mathcal{RS}||_{\text{max}} + ||\mathcal{RS}||_{p_s}\text{max} \). Theorem B.1 guarantees that any object with text relevance no greater than \( \text{ts}_{sz} \) and having the distance from \( q \) no smaller than \( ss_{sz} \) cannot affect the safe zone \( \mathcal{T}^k(\mathcal{RS}) \).

**Theorem B.1:** Consider the non-influence object \( p_s \in (\mathcal{D} \setminus \mathcal{RS} \setminus I) \), such that \( \forall p \neq p_s \land p \in (\mathcal{D} \setminus \mathcal{RS} \setminus I) \) and \( \text{bord}_{\text{min}}(\mathcal{RS}, p_s) < \text{bord}_{\text{min}}(\mathcal{RS}, p) \). Let \( \text{ts}_{sz} = tr_q(p_s) \) and \( ss_{sz} = ||\mathcal{RS}||_{\text{max}} + ||\mathcal{RS}||_{p_s}\text{max} \). Object \( p \) cannot affect the safe zone \( \mathcal{T}^k(\mathcal{RS}) \) if \( tr_q(p) \leq \text{ts}_{sz} \) and \( ||q|| \geq ss_{sz} \).

**Proof:** Since \( p_s \) is a non-influence object, it cannot affect the safe zone, i.e., \( \forall p_j \in \mathcal{RS} \text{bord}_{\text{min}}(p^*_j, p_s) \geq \tau_j \). Consider an object \( p \) such that \( tr_q(p) \leq \text{ts}_{sz} = tr_q(p_s) \) and \( ||q|| \geq ss_{sz} = ||\mathcal{RS}||_{\text{max}} + ||\mathcal{RS}||_{p_s}\text{max} \). Applying the triangle inequality, for any result object \( p^*_j \in \mathcal{RS} \), we have \( ||q|| + ||p^*_j|| > ||q_p|| \geq ||\mathcal{RS}||_{\text{max}} + ||\mathcal{RS}||_{p_s}\text{max} \). Thus, we derive \( ||q_p|| > ||p^*_j|| \). Since \( \text{bord}_{\text{min}}(p^*_j, p) = ||q|| + ||p^*_j|| - tr_q(p^*_j)/tr_q(p^*_j) + tr_q(p) > ||p^*_j|| ||q_p||/(tr_q(p^*_j) + tr_q(p_s)) = \text{bord}_{\text{min}}(p^*_j, p_s) > \tau_j \), object \( p \) cannot affect the safe zone.

Combining the text score intervals and the spatial score intervals derived from the top-k result and the safe zone, we have verification sets

\[
\mathcal{V}_{ST} = T_k \cup T_{sz} = [ts, +\infty)
\]

and

\[
\mathcal{V}_{S} = S_k \cup S_{sz} = [0, ss],
\]

where \( ts = \min\{ts_k, ts_{sz}\} \) and \( ss = \max\{ss_k, ss_{sz}\} \). The verification sets for the sorted lists of coordinates are the coordinate intervals

\[
\mathcal{V}_{SX} = [qx - ss, qx + ss]
\]

and

\[
\mathcal{V}_{SY} = [qy - ss, qy + ss],
\]

which cover the \( x \)-coordinates and \( y \)-coordinates, respectively. The coordinates of \( q \) are represented as \( (qx, qy) \).

As shown in Figure 18, the shaded circle is \( \mathcal{V}_{ST} \). The \( \mathcal{V}_{SX} \) and \( \mathcal{V}_{SY} \) are the bold intervals on the \( x \) and \( y \) axes.

**Fig. 18. Example Spatial Score Interval**

To compose the verification set \( \mathcal{V}_{w_i} \) for the posting list of each query keyword \( w_i \) in the inverted index, the state-of-the-art algorithm Threshold with No Random Access (TNRA) [10] is applied on the inverted index to retrieve the objects whose text relevancies fall inside \( \mathcal{V}_{ST} \). When the TNRA algorithm stops, the posting list of each query keyword has a cut-off value \( cv_i \) that indicates all the objects whose text relevancies fall inside \( \mathcal{V}_{ST} \) have been obtained, i.e., all the objects after the cut-off values are outside \( \mathcal{V}_{ST} \).

Then the verification set on the posting list of word \( w_i \) is:

\[
\mathcal{V}_{w_i} = [cv_i, +\infty).
\]